Effect of the turbulent pumping of the magnetic flux on the predictability of the solar cycle

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Modeling the Grand Minima of solar activity using a flux transport dynamo model

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Maunder minimum period = 1645 to 1715 (Eddy, 1976; Foukal, 1990; Wilson, 1994)

It is a real phenomenon! (Sokoloff & Nesme-Ribes 1994; Hoyt & Schatten 1996)
Characteristics of Maunder Minimum

Hemispheric asymmetry  (Sokoloff & Nesme-Ribes 1994)

Butterfly diagram of sunspot
Cyclic solar activity (Schwab cycle) was continued. (Beer et al. 1998)

But period was longer


From Usoskin, Solanki & Kovaltsov (2007) – 27 grand minima in the last 11,000 years!
Motivation

How grand minima are produced?

Can we model the Maunder minimum or any grand minimum using a dynamo model?

If so, then how frequently it produces grand minima?
Towards flux transport dynamo model

\[ \frac{\partial B}{\partial t} = \nabla \times (\mathbf{V} \times B - \lambda \nabla \times B), \]

with \[ \nabla . B = 0 \]

\[ \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \rho g + \frac{(\nabla \times B)}{\mu_0} \times B - 2\rho \Omega \times \mathbf{v} + 2\nabla . \nu \rho \mathbf{S}, \]

\[ \frac{dp}{dt} = -\rho \nabla . \mathbf{V}, \]

\[ \rho \frac{de}{dt} = -p \nabla . \mathbf{v} + \nabla . \left( \frac{\lambda_{\text{rad}}}{C_V} \mathbf{V} e + 2\nu \rho S^2 + \frac{\lambda}{\mu_0} (\nabla \times B)^2 \right), \]

Kinematic model
Mean-field model  
(Parker 1955; Steenbeck, Krause & Radler 1966)

\[ v = \bar{v} + v', \quad B = \bar{B} + B', \text{ with } \bar{B}' = 0 \text{ and } \bar{v}' = 0 \]

Mean-field induction equation:

\[ \frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{V} \times \bar{B}) + \nabla \times \epsilon + \lambda \nabla^2 \bar{B} \]

\[ \epsilon = \bar{v}' \times \bar{B}' \]

\[ \epsilon = \alpha \bar{B} - \beta \nabla \times \bar{B} \]

where, \[ \alpha = -\frac{1}{3} \bar{v}' \cdot (\nabla \times \bar{v}') \tau \]

and \[ \beta = \frac{1}{3} \bar{v}' \cdot \bar{v}' \tau \]

\[ \frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{V} \times \bar{B}) - \nabla \times (\alpha \bar{B}) + (\lambda + \eta) \nabla^2 \bar{B} \]
Axisymmetric dynamo model

\[ B = B(r, \theta) e_\phi + \nabla \times [A(r, \theta) e_\phi] \]

Velocity field:
\[ \Omega (r, \theta) \sin \theta e_\phi + \nu r e_r + \nu \theta e_\theta \]

Substitute in
\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \nabla \times (\alpha \mathbf{B}) + (\lambda + \eta) \nabla^2 \mathbf{B} \]
Mean-field dynamo equations

Toroidal field evolution:
\[
\frac{\partial B}{\partial t} + \frac{1}{r} \left[ \frac{\partial}{\partial r} \left( rv_r B \right) + \frac{\partial}{\partial \theta} \left( v_\theta B \right) \right] = \eta_t (\nabla^2 - \frac{1}{s^2}) B + s(B \cdot \nabla) \Omega
\]

Poloidal field evolution:
\[
\frac{\partial A}{\partial t} + \frac{1}{s} (v \cdot \nabla) (sA) = \eta_t (\nabla^2 - \frac{1}{s^2}) A + \alpha B
\]

Source terms
1. The Mean Field $\alpha$-effect or classical $\alpha$-effect

- Buoyantly rising toroidal field is twisted by helical turbulent convection, creating loops in the poloidal plane
- The small-scale loops diffuse to generate a large-scale poloidal field

\textit{$\alpha$-effect – works only if $B$ is not very strong}
1. The Mean Field $\alpha$-effect or classical $\alpha$-effect

$\alpha$-effect – works only if $B$ is not very strong


Not self excited!

Observationally verified
Babcock-Leighton process

Depends strongly on average **tilt angle** --- involves randomness

( caused by convective turbulence – Longcope & Fisher 1996

Or the inflow of the active regions)

Supported by Dasi-Espuig et al. (2010)
Kitchatinov & Olemskoy (2011)

Charboneau et al. (2004) - intermittencies like grand minima.
Modelling a Maunder minimum

Assumption: Poloidal field drops to 0.0 and 0.4 of its average value in the two hemispheres

Choudhuri & Karak (2009)
Another source of randomness in flux transport dynamo

Fluctuations in Babcock-Leighton process of poloidal field generation

Variation in meridional circulation
Variation of meridional circulation

**Indirect evidences**

Wang et al. (2002)

Hathaway et al. (2003)


Passos & Lopes (2008)

Georgieva & Kirov (2010)

Karak (2010)
Fluctuation of the meridional circulation

Period $\propto \frac{1}{v_0^{0.89}}$
(Dikpati & Charbonnea 1999)

Period $\propto \frac{1}{v_0^{0.88}}$
(Yeates, Nandy & Mackey 2008)
Fluctuation of the meridional circulation

Period $\propto \frac{1}{v_0^{0.89}}$

(Dikpati & Charbonnea 1999)

Period $\propto \frac{1}{v_0^{0.88}}$

(Yeates, Nandy & Mackey 2008)
Sufficiently large decrease in meridional circulation can cause grand minimum

Karak (2010)
Physics of earlier result (Yeates, Nandy & Mackey 2008)

\[ \frac{\partial B}{\partial t} + \ldots = \eta_t \left( \nabla^2 - \frac{1}{s^2} \right) B + s \left( B_p \cdot \nabla \right) \Omega \]

Meridional circulation

More time to induct toroidal field

Stronger cycle

More time for the diffusion

Weaker cycle

This is the case in our model!
Sufficiently large decrease in meridional circulation can cause grand minimum

Periods during grand minimum should be longer!

Karak (2010)
Observational evidences of the longer periods during grand minima


Modeling Maunder minimum

Large decrease of the poloidal field

Large decrease of the meridional circulation

Maunder-like grand minimum

Karak (2010)
Observational data: (Usoskin et al. 2007)

Grand minima of length ~ 20 yrs

Grand minima of >20 yrs
How to find out the strength of the meridional circulation in past?

In flux transport dynamo:

\[
\text{Period} \propto \frac{1}{v_0^{0.89}}
\]
(Dikpati & Charbonneua 1999)

\[
\text{Period} \propto \frac{1}{v_0^{0.88}}
\]
(Yeates, Nandy & Mackey 2008)
Meridional circulation of last 28 cycle
Distribution of Meridional circulation
How to find the strength of the poloidal field?

Polar field is a measure of the next sunspot cycle!

Using Dynamo Theory to Predict

The Sunspot Number During Solar Cycle 21

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Abstract. On physical grounds it is suggested that the sun's polar field strength near a solar minimum is closely related to the following cycle's solar activity. Four methods of estimating the sun's polar magnetic field strength near solar minimum are employed to provide an estimate of cycle 21's yearly mean sunspot number at solar minimum of 140 \pm 20. We think of this estimate as a first order attempt to predict the cycle's activity using one parameter of physical importance based upon dynamo theory.

Polar Field Strength

Estimates of the polar magnetic field near sunspot minimum may be obtained from shape of the corona at the time of solar or by the amount of flattening of the "wake current sheet" at 1AU as obtained from planetary magnetic field measurements and in accordance with the methods of Rosenberg Coleman (1969). A further and more direct estimate of polar field strength is obtained observing the number of polar faculae.
How to find out the strength of the poloidal field?

Assume a perfect correlation between the **peak sunspot number** and the **polar field of the previous cycle**
How to find out the strength of the poloidal field?

Assume a perfect correlation between the peak sunspot number and the poloidal field of the previous cycle.

Sunspot number

Poloidal field strength
Distributions of Meridional circulation and poloidal field strength

\[ P(\gamma, v_0) dv_0 d\gamma = \frac{1}{\sigma_v \sqrt{2\pi}} \exp \left[ -\frac{(v_0 - \bar{v}_0)^2}{2\sigma_v^2} \right] \times \frac{1}{\sigma_\gamma \sqrt{2\pi}} \exp \left[ -\frac{(\gamma - 1)^2}{2\sigma_\gamma^2} \right] d\gamma dv_0 \]
\[ P(\gamma, v_0) d\gamma dv_0 = \frac{1}{\sigma_v \sqrt{2\pi}} \exp \left[ -\frac{(v_0 - \bar{v}_0)^2}{2\sigma_v^2} \right] \times \frac{1}{\sigma_\gamma \sqrt{2\pi}} \exp \left[ -\frac{(\gamma - 1)^2}{2\sigma_\gamma^2} \right] d\gamma dv_0 \]

\[ \int \int p(\gamma, v_0) d\gamma dv_0 \]

**Theoretical value**

\[ = 1.8\% \quad (\pm 0.6) \]

**Observational value**

\[ = 2.7\% \]
Results of simulation of grand minima

We get 20–28 grand minima in 11,000 years
Observational value = 27 (Usoskin et al. (2007)

(Choudhuri & Karak 2012)
Waiting times of grand minima based on 27 grand minima in last 11,400 years reported by Usoskin et al. (2007) is also exponential -- governed by stationary memoryless stochastic processes whereas the observed distribution of durations is not so conclusive.
Conclusions

- Grand minima are probably caused by the fluctuation in the poloidal field and the fluctuation in the meridional circulation.

- By measuring the fluctuations of these we can model the observed frequency of grand minima using flux transport dynamo model.

- Recovery from grand minima is less understood.
During grand minima Babcock-Leighton mechanism may no work as there are no sunspots???
Alpha effect proposed by Parker (1955) and Steenbeck, Krause & R. Adler (1969) is a good candidate!
What about grand maxima

What about grand maxima

What about grand maxima

What about grand maxima

What about grand maxima

What about grand maxima

What about grand maxima

What about grand maxima
Thanks
\( \alpha \)-quenching:

\[ \alpha = \frac{\alpha_0}{1 + |B|^2} \]

; \[ \frac{\partial A}{\partial t} + \ldots = \eta \rho (\nabla^2 - \frac{1}{s^2}) A + \alpha B \]


Has a stabilizing effect instead of producing irregularities

Charbonneau, St-Jean & Zacharias (2005), Charbonneau, Beaubien & St-Jean (2007) – The odd-even effect (Gnevyshev-Ohl rule) may be due to period doubling just beyond bifurcation point

Weiss, Cattaneo & Jones (1984) found chaos in some highly truncated models

Beer, Tobias & Wiess (1998) grand minima are due to chaotic nature of the dynamo