Magnetic flux emergence with differential rotation in compressible shells

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Description of the project

• Simulate rising magn. flux tubes for diff. type of stars.
  – compressible stratified convection zone
  – turbulent convection
  – realistic differential rotation (forced)
• First step: solar case for testing our model and compare with literature.
• Second step: to apply the model to red giants.
What do we need?

- We need the following ingredients:
  - Stellar model for the *stratification*
  - A mean field model of angular momentum transport for the *realistic differential rotation*
  - A dynamo model (kinematic) for the *magn. distribution*
  - A parallel compressible MHD code with Adaptive Mesh Refinement in spherical coordinates for *convection* and the *dynamical evolution of the magn. flux*
What are we doing?

First step:
- 2D fully compressible MHD
- Adiabatically stratified spherical shell
- Forced diff. Rotation
- Rising flux tube with AMR
The model
The model

- Domain, boundary conditions, and initial conditions

Hydrostatic Equilibrium

\[ \frac{\partial P}{\partial r} \hat{r} = -\rho g \]

\[ \frac{\partial T}{\partial r} \hat{r} = -\frac{F_{\text{rad}}}{\kappa} \]

Definition of gravity and radiative flux

\[ g = -\frac{G M_{\odot}}{r^2} \hat{r} \]

\[ F_{\text{rad}} = \frac{L_{\odot}}{4\pi r^2} \hat{r} \]
The model

- Compressible MHD equations

\[ \partial_t(\rho) + \nabla \cdot (\rho \mathbf{u}) = 0, \]
\[ \partial_t(m) + \nabla \cdot \left[ \rho \mathbf{v} \mathbf{v} + P_{\text{tot}} I - \frac{1}{\mu_0} \mathbf{B} \mathbf{B} \right] = -\rho \mathbf{g} + \mathbf{f}_{cc} + \mathbf{f}_{dr} \]
\[ \partial_t(e) + \nabla \cdot \left[ (e + P_{\text{tot}}) \mathbf{v} - \frac{1}{\mu_0} (\mathbf{v} \cdot \mathbf{B}) \mathbf{B} \right] = (-\rho \mathbf{g} + \mathbf{f}_{CC} + \mathbf{f}_{DR}) \mathbf{v} - \mathbf{F}_{\text{rad}} \]
\[ \partial_t(\mathbf{B}) - \nabla \times (\mathbf{u} \times \mathbf{B}) = 0, \]

- Dimensionless system (Käpylä et al. 2010a)

\[ \tilde{\rho}_{\text{top}} = G \tilde{M}_{\odot} = \tilde{R}_{\odot} = \tilde{c} \tilde{p} = 1 \]
The model

- **Forcing term**
  - We do not simulate self consistently diff. Rotation.
  - We add a force in the $\phi$ direction which enforce the diff. rotation profile.

\[ f_{DR} = -\frac{\langle v_\phi \rangle - v_{\text{diff}}}{\tau_{\text{relaxation}}} \]

- **Relaxation time**

\[ \tau_{\text{relaxation}} = \tau_0 [1 + \alpha \beta] \]

Physically motivated by the fact that magn. Field suppresses Reynolds stress
## Results

- **The path of thought:**

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Buoy.</th>
<th>Magn. T.</th>
<th>Cor. F.</th>
<th>Solar DF</th>
<th>Cyl. DF</th>
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</thead>
<tbody>
<tr>
<td>Without rotation</td>
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<tr>
<td>With solid body rotation</td>
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<tr>
<td>With enforced solid body rotation</td>
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<tr>
<td>With solar-like diff. rotation</td>
<td>x</td>
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<td>x</td>
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<td>x</td>
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<tr>
<td>With cylindrical diff. rotation</td>
<td>x</td>
<td>x</td>
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</tbody>
</table>
Results

- Without rotation

Agrees with Choudhuri, & Gilman, 1987

$\beta$: 0.023
$\tilde{\Omega}$: 0.757
Results

- Solid Body rotation
Results

- Solid body rotation enforced by the forcing term

Agrees with Fan et al. 1994
Caligari et al. 1995
Results

- Effect of the forcing term
Results

- Solar-like diff. rotation
• With Cylindrical dif

\[ \beta: 0.023 \]
\[ \hat{\Omega}: 0.757 \]
Results

- Effect of diff. Rotat
Discussion

• These are just preliminary work
  – Parameter study for different initial latitudes and magn. field strength.
  – Apply this model on Giants
  – Add more physics
    • Convection
    • 3D
    • Turbulence?

Convection similar to Käpylä 2010a
Final simulation similar to Jouve & Brun 2006/2013
Conclusion

• We have all the ingredients we need for designing an advanced model.

• We choose not to simulate diff. rot. but to force it.

• We obtain confident results in 2D without convection.

• We are now going to study giants, to go to 3D, and add convection.
Questions, Comments, & Critics

Thanks for your attention
Conclusion

• We have all the ingredients we need for designing an advanced model and learn from it.

• We choose not to simulate diff. rot. but to force it.

• We obtain confident results in 2D without convection.

• We are now going to study giants, to go to 3D, and add convection.
References

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Kätälä, Korpi, Brandenburg, Mitra, Tavakol 2010 AN
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