Role of polymers in the mixing of Rayleigh-Taylor turbulence

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Outline of the talk

1) Rayleigh-Taylor turbulence: quick view to the Newtonian case

2) Viscoelastic Rayleigh-Taylor turbulence
   • Effect of Polymers on mixing
   • Heat transfer enhancement
Rayleigh-Taylor instability

Instability on the interface of two fluids of different densities with relative acceleration.

Rayleigh (1883): unstable stratification in gravitational field
Taylor (1950): generalization to all acceleration mechanisms

Atwood number \( A \equiv \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \)

Linear analysis: exponential growth with rate \( \sqrt{Agk} \)
Rayleigh-Taylor turbulence (2D case)

\[
\partial_t u + u \cdot \partial u = -\partial p + \nu \partial^2 u - \beta g T
\]

\[
\partial_t T + u \cdot \partial T = \kappa \partial^2 T
\]

Buoyancy balances inertia at all scales

- **Temperature cascades toward small scales**
- **Velocity: inverse energy cascade (flux not constant)**

small scale fluctuations follow **Bolgiano** scaling

\[
\delta_r u(t) \sim (Ag)^{2/5} t^{-1/5} r^{3/5}
\]

\[
\delta_r T(t) \sim \theta_0 (Ag)^{-1/5} t^{-2/5} r^{1/5}
\]

M. Chertkov, PRL 91 (2003)

A. Celani, A. Mazzino, L. Vozella, PRL 96 (2006)

Biferale et al, PF 22 (2011)
Rayleigh-Taylor turbulence (3D case)

\[ \partial_t u + u \cdot \partial u = -\partial p + \nu \partial^2 u - \beta g T \]

\[ \partial_t T + u \cdot \partial T = \kappa \partial^2 T \]

- **Temperature cascades toward small scales**
- **Velocity:** direct energy cascade (constant flux)

Buoyancy is negligible at small scales

M. Chertkov, PRL 91 (2003)
Rayleigh-Taylor turbulence (3D case)

\[ \partial_t u + u \cdot \nabla u = -\partial p + \nu \nabla^2 u - \beta g T \]

\[ \partial_t T + u \cdot \nabla T = \kappa \nabla^2 T \]

- Temperature cascades toward small scales
- Velocity: direct energy cascade (constant flux)

Buoyancy is negligible at small scales

M. Chertkov, PRL 91 (2003)


Small scale fluctuations follow Kolmogorov-Obukhov scaling

\[ \delta_r u(t) \sim (Ag)^{2/3} t^{1/3} r^{1/3} \]

\[ \delta_r T(t) \sim \theta_0 (Ag)^{-1/3} t^{-2/3} r^{1/3} \]
From Newtonian to viscoelastic Rayleigh Taylor
Drag reduction (Toms 1949)
polymers can reduce the friction drag in a pipe flow up to 80%

Dilute polymer solutions
polymers are stretched by velocity gradients, they can store and dissipate elastic energy

Viscoelastic models: Oldroyd-B
viscous + elastic stress tensor

\[
\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \Delta \mathbf{u} - \beta g T + \frac{2\nu \eta}{\tau_p} \nabla \cdot \boldsymbol{\sigma}
\]

\[
\partial_t T + \mathbf{u} \cdot \nabla T = \kappa \Delta T
\]

\[
\partial_t \boldsymbol{\sigma} + \mathbf{u} \cdot \nabla \boldsymbol{\sigma} = (\nabla \mathbf{u})^T \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} \cdot (\nabla \mathbf{u}) - \frac{2}{\tau_p} (\boldsymbol{\sigma} - I)
\]

\[
\tau_p = \frac{\mu R_0^3}{k_B T}
\]

Conformation tensor

\[
\sigma_{\alpha\beta} \equiv \langle R_\alpha R_\beta \rangle
\]

Zimm relaxation time
Viscoelastic Rayleigh-Taylor: linear stability

Linear stability analysis on the Oldroyd-B perturbation growth-rate $\alpha$

$$
(\alpha \tau_p)^3 + 2(\alpha \tau_p)^2 (1 + \nu k^2 \tau_p) + \alpha \tau [4\nu (1 + \eta) k^2 \tau_p - \omega^2 \tau_p^2] - 2\omega^2 \tau_p^2 = 0
$$

zero-shear polymer contribution to total viscosity

Zimm relaxation time

$$
\omega = \sqrt{A g k}
$$

Relevant asymptotics:

1) $\tau_p \rightarrow 0$ (Newtonian case)  \hspace{1cm} \alpha_0 = -\nu (1 + \eta) k^2 + \sqrt{\omega^2 + [\nu (1 + \eta) k^2]^2}$

2) $\tau_p \rightarrow \infty$ (pure solvent Newt. case)  \hspace{1cm} \alpha_\infty = -\nu k^2 + \sqrt{\omega^2 + \nu^2 k^4} \geq \alpha_0$
Viscoelastic Rayleigh-Taylor: linear stability

From implicit differentiation, \( \alpha(\tau_p) \) is monotonic.

\[ \alpha_{\infty} \geq \alpha_0 \]
Viscoelastic Rayleigh-Taylor: linear stability

From implicit differentiation, \( \alpha(T_p) \) is monotonic.

Because \( \alpha_\infty \geq \alpha_0 \)

The instability growth-rate increases with the elasticity.

Viscoelastic Rayleigh-Taylor: linear stability

From implicit differentiation

\[ \alpha(\tau_p) \] is monotonic

Because \[ \alpha_\infty \geq \alpha_0 \]

The instability growth-rate increases with the elasticity

?? Similar speed-up in turbulence ??

Heuristics in the “passive case” (2D)

From mean field arguments:

2D RT obeys Bolgiano-Obukhov phenomenology: \[ \delta_r u(t) \sim (Ag)^{2/5} t^{-1/5} r^{3/5} \]

Energy flux flows toward large scales at non-constant flux \[ \epsilon \sim (Ag)^{6/5} t^{-3/5} r^{4/5} \]

Viscous Kolmogorov scale: \[ \eta \sim \frac{\nu}{\delta_\eta u} = \nu^{5/8} t^{1/8} (Ag)^{-1/4} \]

Kolmogorov time scale: \[ \tau_\eta \sim \frac{\eta}{\delta_\eta u} = \nu^{1/4} t^{1/4} (Ag)^{-1/2} \]
Heuristics in the “passive case” (2D)

Weissemberg number:

\[ Wi \equiv \frac{\tau_p}{\tau_\eta} \sim t^{-1/4} \]

No coil-stretch transition is expected in 2D

Lumley scale:

\[ \frac{l_L}{\delta l_L u} \sim \tau_p \]

\[ l_L \sim \tau_p^{5/2} (Ag) t^{-1/2} \]

At late times \( l_L \ll \eta \)

polymers: only contribute to viscosity renormalization
Heuristics in the “passive case” (3D)

From mean field arguments:

3D RT obeys K41 phenomenology: \( \delta_r u(t) \sim \epsilon^{1/3} r^{1/3} \)

Energy flux flows toward small scales at constant flux: \( \epsilon \sim (Ag)^2 t \)

Viscous Kolmogorov scale: \( \eta \sim \frac{\nu}{\delta_\eta u} \sim \nu^{3/4} \epsilon^{-1/4} \sim \nu^{3/4} (Ag)^{-1/2} t^{-1/4} \)

Kolmogorov time scale: \( \tau_\eta \sim \frac{\eta}{\delta_\eta u} = \nu^{1/2} t^{-1/2} (Ag)^{-1} \)
Heuristics in the “passive case” (3D)

Weissemberg number:

\[ Wi \equiv \frac{\tau_p}{\tau_\eta} \sim t^{1/2} \]

Coil-stretch transition is expected in 3D

Lumley scale:

\[ \frac{l_L}{\delta_{l_L} u} \sim \tau_p \quad l_L \sim \tau_p^{3/2} (Ag)t^{1/2} \]

\[ \eta \ll l_L \ll L \]

well within the inertial range of scales
• Parallel pseudo-spectral code

• Second-order Runge-Kutta temporal scheme

• Interface temperature initial perturbation: 10% white noise

• Simulations halted at $L(t) \sim 80\%$ $Ly$

Main parameters:

<table>
<thead>
<tr>
<th>Run</th>
<th>$N_{x,y}$</th>
<th>$N_z$</th>
<th>$L_{x,y}$</th>
<th>$L_z$</th>
<th>$\theta_0$</th>
<th>$\beta g$</th>
<th>$\nu = \kappa$</th>
<th>$\kappa_p$</th>
<th>$\eta$</th>
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<td>1024</td>
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<td>4$\pi$</td>
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<tr>
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<td>2$\pi$</td>
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</tr>
</tbody>
</table>
PDF of polymers elongation

Coil-stretch transition at $t \sim \tau$

An intrinsic exp cutoff emerges for polymers elongation
Energy balance in viscoelastic RT

Kinetic and elastic energy

\[ E = K + \Sigma = \frac{1}{2}\langle u^2 \rangle + \left( \nu \eta / \tau_p \right) \langle tr \sigma \rangle \]

Produced from potential energy

\[ P = -\beta g \langle zT \rangle \]

Effects of polymers:

- Speed-up of potential energy consumption
- Increase of kinetic energy
- Reduction of viscous dissipation (not shown)

“Drag reduction”

Benzi, De Angelis, Govindarajan and Procaccia, PRE (2003);
De Angelis, Casciola, Benzi and Piva, JFM (2005)
Energy balance in viscoelastic RT

Kinetic and elastic energy

\[
E = K + \Sigma = \frac{1}{2} \langle u^2 \rangle + \left( \nu \eta / \tau_p \right) \langle tr \sigma \rangle
\]

produced from potential energy

\[
P = -\beta g \langle zT \rangle
\]

\[
- \frac{dP}{dt} = \beta g \langle wT \rangle = \frac{dE}{dt} + \epsilon_\nu + \frac{2\nu \eta}{\tau_p^2} \left[ \langle tr \sigma \rangle - 3 \right]
\]

Effects of polymers:

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“Drag reduction”

Benzi, De Angelis, Govindarajan and Procaccia, PRE (2003); De Angelis, Casciola, Benzi and Piva, JFM (2005)
Quantification of Drag Reduction (DR)

**DR** = loss of potential energy/plumes kinetic energy

Potential energy: \( P = -\beta g \langle zT \rangle \)

Kinetic energy: \( K \sim \frac{1}{2}[\dot{h}(t)]^2 \)  

(Fermi and von Neumann, 1955)

Assuming linear vertical profile for \( T \): \( P(t) \sim -1/6 Ag h(t) \)

\[
f \equiv \frac{\Delta P}{K} = \frac{1}{3} Ag \frac{h}{\dot{h}^2} = \frac{1}{12\alpha} \quad (h(t) = \alpha Ag t^2)
\]

\( \alpha \) is thus related to \( f \)
Quantification of Drag Reduction

22 % of DR for run B
30 % of DR for run C
Quantification of Drag Reduction

Energy spectra

Increasing Weißeberg

- More efficient conversion from potential to kinetic energy
- Reduced energy transfer to small scales (reduced dissipation)

Polymers:

Similarities with isotropic homogeneous turbo: Benzi, De Angelis, Govindarajan and Procaccia, PRE (2003); De Angelis, Casciola, Benzi and Piva, JFM (2005)
Faster growth of mixing layer $h(t)$

More efficient mass transfer
Faster growth of mixing layer $h(t)$

a) Faster growth of mixing layer $h(t)$
   - More efficient mass transfer

Larger temperature variance $\sigma_T$

b) Reduced mixing efficiency at small scale
Heat transfer enhancement

\[ \text{Ra} \equiv \frac{\beta g \theta_0 h^3}{\kappa \nu} \]

\[ \text{Re} \equiv \frac{u_{\text{rms}} h}{\nu} \]

\[ \text{Nu} \equiv \frac{\text{Corr}(wT) h w_{\text{rms}}}{\kappa} \]

Nusselt increases both vs time and vs Rayleigh

because: Corr(wT), w_{\text{rms}}, h increase

w.r.t. RB convection:
agreement (Benzi et al, PRL 2010); disagreement (Ahlers and Nikolaenko, PRL 2010)
Main Conclusions

- Speed-up of Rayleigh-Taylor instability due to Polymers (linear analysis)
- Polymers increase the rate of large-scale mixing and reduce small-scale mixing in the fully developed turbulence stage
- Many analogies with DR in homogeneous isotropic turbulence
- Heat transport enhancement

Perspectives

- Extension to elastic fibers
- Extension to immiscible Newtonian fluids
Details on the viscoelastic RT:

Rayleigh–Taylor instability in a viscoelastic binary fluid
* G. Boffetta, A. Mazzino, S. Musacchio and L. Vozella

Polymer Heat Transport Enhancement in Thermal Convection: The Case of Rayleigh-Taylor Turbulence
* G. Boffetta, A. Mazzino, S. Musacchio and L. Vozella

Effects of polymer additives on Rayleigh-Taylor turbulence
* G. Boffetta, A. Mazzino and S. Musacchio
Relevance of Rayleigh-Taylor instability

Many applications in natural phenomena and technological problems:

- supernova explosion
- acceleration mechanism for thermonuclear flame front
- atmospheric physics (mammatus clouds)
- solar corona heating
- inertial confinement fusion
- etc.
Viscoelastic Rayleigh-Taylor turbulence

Newtonian

Viscoelastic
Faster and more coherent plumes

Horizontal velocities $u_{\text{rms}}$ are depleted
Vertical velocities $w_{\text{rms}}$ are enhanced

$R_u, R_w$ half width vel. correlat.
Faster and more coherent plumes

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