Neutron-proton pair coupling from a shell-model perspective

Chong Qi
Royal institute of Technology (KTH), Stockholm

Background and motivation
Seniority and systems with identical particles in a single-$j$ orbital
Neutron-proton correlation from the binding energy
Neutron-proton spin-aligned pair coupling

>=2 slides can be mistaken.
The greatest challenge is to understand the complicated full wave function:
How to filter out the relevant components
'Physics’ in a single-particle orbital?

- The nuclear 'single-particle' state has a complicated nature;
- **Strong isovector pairing correlation**;
- **Strong np quadrupole-quadrupole correlation**;

**On single nucleon wave functions in nuclei**

Igal Talmi

*The Weizmann Institute of Science, Rehovot 76100 Israel*

**Abstract.** The strong and singular interaction between nucleons, makes the nuclear many body theory very complicated. Still, nuclei exhibit simple and regular features which are simply described by the shell model. Wave functions of individual nucleons may be considered just as model wave functions which bear little resemblance to the real ones. There is, however, experimental evidence for the reality of single nucleon wave functions. There is a simple method of constructing such wave functions for valence nucleons. It is shown that this method can be improved by considering the polarization of the core by the valence nucleon. This gives rise to some rearrangement energy which affects the single valence nucleon energy within the nucleus.

I. Talmi, AIP Conf. Proc. 1355, 121 (2011)

**Non-observability of Spectroscopic Factors**

B.K. Jennings*

TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, V6T 2A3

(Dated: February 21, 2011)

**Abstract**

The spectroscopic factor has long played a central role in nuclear reaction theory. However, it is not an observable. Consequently it is of minimal use as a meeting point between theory and experiment. In this paper the nature of the problem is explored. At the many-body level, unitary transformations are constructed that vary the spectroscopic factors over the full range of allowed values. At the phenomenological level, field redefinitions play a similar role and the spectroscopic factor extracted from experiment depend more on the assumed energy dependence of the potentials than on the measured cross-sections. The consistency conditions, gauge invariance and Wegmann’s theorem play a large role in these considerations.

B.K. Jennings, arXiv:1102.3721

• **Nuclear physics is an emergent phenomenon** (Philip Warren Anderson).
• **Nobody of theoretical physicists could have predicted the existence of a nucleus from first principles.** (DJ Rowe & JL Wood)
The coupling of few nucleons

Seniority and systems with identical particles in a single-\(j\) orbital

Neutron-proton correlation from the binding energy

Neutron-proton spin-aligned pair coupling
General properties of the effective interaction

- **Isovector (T=1):** $J=0,2,\ldots,2J-1$, $J=0$ term attractive (*pairing*), others close to zero
- **Isoscalar (T=0):** $J=1,3,\ldots,2j$, strongly attractive (mean field)
  - The $J=1$ and $2j$ terms are the most attractive ones.
  - $L=0, J=1$ pairing
  - The aligned pair was not much studied

\[ \cos \theta_{12} = \frac{J(J+1)}{2j(j+1)} - 1 \]

*J.P. Schiffer and W.W. True, Rev.Mod.Phys. 48,191 (1976)*
Seniority coupling scheme with realistic interaction

\[ |g.s.\rangle = |\nu = 0; J = 0\rangle = (P_j^+)^n/2 |\Phi_0\rangle \]

\[ |\nu = 2; JM\rangle = (P_j^+)^{(n-2)}/2 A^+(j^2 JM)|\Phi_0\rangle \]

Energy levels of Oh_{11/2} protons in N=82 isotones

Energy levels of Og_{9/2} protons in N=50 isotones

9j symbol

One can construct a nonorthogonal, unnormalized, and overcomplete two-pair basis

\[ |(j_1 j_3) J_{13} (j_2 j_4) J_{24}; JM\rangle = \sum_{J_{12}, J_{34}} \hat{J}_{13} \hat{J}_{24} \hat{J}_{12} \hat{J}_{34} \]

\[ \times \begin{cases} j_1 & j_2 & J_{12} \\ j_3 & j_4 & J_{34} \\ J_{13} & J_{24} & J \end{cases} |(j_1 j_2) J_{12} (j_3 j_4) J_{34}; JM\rangle . \]

- For j<9/2, seniority is (automatically) conserved!
- The rotationally invariant interaction has to satisfy \([(2j - 3)/6]\) linear constraints to conserve seniority.
Partial dynamic conservation of seniority symmetry

\[ |J_1 J_2 J \rangle = - \sum_{J_1' J_2'} \hat{j}_1 \hat{j}_2 \hat{j}_1' \hat{j}_2' X( j j J_1; j j J_2; J_1' J_2') |J_1' J_2' J \rangle, \]

FIG. 1. The spectrum of four particles in a single-\( j \) shell \((j = \frac{1}{2}, H = -Q \cdot Q, \) energies are in arbitrary units\). Part \(a\), the shell-model calculation; \(b\), the GPFM calculation.
Seniority coupling for many shells

- Shell model calculations with a pairing Hamiltonian.
- The physical vector only spans the v=0 subspace.
- There are as many independent solutions as states in the v=0 space.

\[ \langle \{ s_j \}, \ldots N_j + 2, \ldots N_{j'} - 2, \ldots | \quad \times H | \{ s_j \}, \ldots N_j, \ldots N_{j'}, \ldots \rangle \]

\[ = \frac{G_{j j'}}{4} \left[ (N_{j'} - s_{j'}) (2 \Omega_j - s_{j'} - N_{j'} + 2) \right. \]

\[ \left. \times (2 \Omega_j - s_j - N_j) (N_j - s_j + 2) \right]^{1/2}. \]

10^2 in seniority v=0 space
Different facets of the nucleus
Spin inversion in $^{103}$Sn with respect to that of $^{101}$Sn pseudospin symmetry


Neutron-proton coupling

Neutron-proton correlation from the binding energy

Neutron-proton spin-aligned pair coupling
The spin trap isomers:

**The 12\(^+\) spin trap in \(^{52}\)Fe**

\[
E_{12}(^{52}\text{Fe}) = \frac{6}{13} \tilde{V}_5 + 3 \tilde{V}_6 + \frac{33}{13} \tilde{V}_7,
\]

\[
E_{10i}(^{52}\text{Fe}) = 0.310 \tilde{V}_3 + 1.429 \tilde{V}_4 + 0.497 \tilde{V}_5 + 1.571 \tilde{V}_6 + 2.193 \tilde{V}_7,
\]

*Q. Qiu, Phys. Rev. C 81, 034318 (2010).*

**The predicted 16\(^+\) spin trap in \(^{96}\)Cd**

\[
E_{16}(^{96}\text{Cd}) = \frac{8}{17} \tilde{V}_7 + 3 \tilde{V}_8 + \frac{43}{17} \tilde{V}_9.
\]

\[
E_{14i}(^{96}\text{Cd}) = 0.307 \tilde{V}_5 + 1.428 \tilde{V}_6 + 0.493 \tilde{V}_7 + 1.572 \tilde{V}_8 + 2.200 \tilde{V}_9.
\]


*Q. Qiu, Phys. Rev. C 81, 034318 (2010).*

The relative positions of these spin traps are sensitive to the strength of the interaction \(V_{J=2j}\).
The energy expression

\[ E_I = C_J^I V_J, \]

\[ C_J^I = \frac{1}{2} \langle \left| \left( a^+ a^+ \right)^J \left( a a \right)^J \right| \rangle \]

The total number of pairs with all spins \( J \) is given by

\[ \sum_J C_J^I = n(n - 1)/2, \]

and

\[ \sum_{J, \text{odd}} C_J^I = \frac{1}{2} \left[ \frac{n}{2} \left( \frac{n}{2} + 1 \right) - T(T + 1) \right], \]

For systems with \( n=4 \) and isospin \( T=0 \), there are three isoscalar pairs and three isovector pairs;

For those with \( T=2 \) (four identical nucleons), we have six isovector pairs.

16\(^{+}\) Spin-Gap Isomer in \(^{96}\text{Cd}\)
Shell model calculations on the alpha formation amplitude in N=Z nuclei.
Seniority with isospin

\[ E = \varepsilon n + \frac{a}{2} n(n-1) + \frac{b}{2} \left[ \mathcal{T}(\mathcal{T} + 1) - \frac{3n}{4} \right] \]
\[ + G \left[ \frac{n-v}{4} (4j+8-n-v) - \mathcal{T}(\mathcal{T} + 1) + s(s+1) \right], \]

Odd-even staggering

\[ E = \varepsilon n + \frac{2a-G}{4} n(n-1) \]
\[ + \frac{b-2G}{2} \left[ \mathcal{T}(\mathcal{T} + 1) - \frac{3n}{4} \right] \]
\[ + (j+1)G(n-v) + G \left[ \frac{v^2}{4} - v + s(s+1) \right], \]

Pairing energy in mass formula \( E_p \propto 2 - v, \)

Fig. 2. (Color online.) Empirical proton-proton (squares) and neutron-neutron (circles) interactions in even-even nuclei extracted from experimental nuclear masses as a function of the mass number \( A \) [23]. The solid symbols denote those in the \( N = Z \) nuclei.
The binding energies of even-even and odd-odd N=Z nuclei are compared. After correcting for the symmetry energy we find that the lowest T=1 state in odd-odd N=Z nuclei is as bound as the ground state in the neighboring even-even nucleus, thus providing evidence for isovector np pairing. However, T=0 states in odd-odd N=Z nuclei are several MeV less bound than the even-even ground states. We associate this difference with the T=1 pair gap and conclude from the analysis of binding energy differences and blocking arguments that there is no evidence for an isoscalar (deuteronlike) pair condensate in N=Z nuclei.
The average proton-neutron interaction

\[ V_{pn}(Z, N) = \frac{1}{4} \left[ B(Z, N) + B(Z - 2, N - 2) - B(Z - 2, N) - B(Z, N - 2) \right], \]

Additional binding for $N=Z$ nuclei

\[ \hat{V} = a + bt_1 \cdot t_2 + GP_0, \]

For even–even nuclei with $n_\pi \neq n_\nu$,

\[ V_{pn} = -\frac{4V_{m;T=1} + 2(V_{m;T=0} - V_{m;T=1})}{4} = \frac{b}{4} - a. \]

in the case of $n_\pi = n_\nu$ (i.e., $N = Z$),

\[ V_{pn} = -\frac{4V_{m;T=1} + 3(V_{m;T=0} - V_{m;T=1})}{4} - \frac{G}{2} = \frac{b}{2} - a - \frac{G}{2}. \]

**odd–odd $N = Z$**

\[ V_{pn}(Z - 1, Z - 1) = B(Z - 1, Z - 1) + B(Z - 2, Z - 2) - B(Z - 1, Z - 2) - B(Z - 2, Z - 1) \]

\[ = \frac{3b}{4} - a. \]

---

**Fig. 4.** (Color online.) Experimental $V_{pn}$ values of even–even $N = Z$ nuclei (filled circles) and the adjacent odd–odd (squares) and odd–$A$ nuclei (triangles). The filled and open triangles correspond to systems with one nucleon subtracted from and added to the even–even nuclei, respectively. The solid line labeled $1^*$ describes the average behavior of $V_{pn}$ in even–even $N \neq Z$ nuclei from **Fig. 1.** $2^*$ and $3^*$ denotes its twice and three time values.
But many N=Z nuclei are deformed

\[ N = Z \]

- QQ correlation induces deformation;
- The np interaction also breaks the seniority in a major way

\[ ^{48}\text{Cr} \]
The $T=0 \langle \alpha \alpha \rangle$ pairing exhibits a rotational like behavior as a function of frequency. The $I_x$ operator may not be the right one for this collective mode.

A.L. Goodman, PRC 63, 044325 (2001)
Nuclei around $^{100}$Sn: $N=Z=50$ shell closures survive

Superallowed Gamow–Teller decay of the doubly magic nucleus $^{100}$Sn

C. B. Hinke$^2$, M. Böhm$^3$, P. Bountachkov$^2$, T. Faestermann$^1$, H. Geissel$^1$, J. Ger$^2$, R. Gernhäuser$^1$, M. Gőrski$^2$, A. Gottardo$^3$, H. Graebe$^2$, J. L. Grégoire$^4$, R. Krücker$^5$, N. Kurz$^6$, Z. Liu$^7$, L. Maier$^1$, F. Nowacki$^3$, S. Pietri$^3$, Zs. Podolyák$^8$, K. Steigl$^1$, K. Streiter$^1$, K. Straub$^1$, H. Weck$^2$, H.-J. Wollersheim$^9$, P. J. Woods$^9$, N. Al-Dahan$^9$, N. Alkhormashi$^9$, A. Atac$^9$, A. Blazhev$^{10}$, N. F. Braun$^{10}$, I. T. Chetkov$^{10}$, T. Davidson$^{11}$, I. Dillmann$^{11}$, C. Domingo-Pardo$^{12}$, P. C. Doorenbos$^{12}$, G. de France$^{12}$, G. F. Ferrell$^{12}$, F. Firan$^{13}$, N. Goel$^{13}$, T. C. Habermann$^{13}$, R. Hölschen$^{13}$, R. Junik$^{13}$, M. Karsiny$^{13}$, A. Kasjnov$^{13}$, I. M. Kojouharov$^{13}$, Th. Krüger$^{13}$, Y. Litvinov$^{13}$, S. Myalski$^{13}$, F. Nebel$^{13}$, N. Nishimura$^{13}$, C. Nociforo$^{13}$, J. Nyberg$^{13}$, A. R. Park$^{13}$, A. Procházková$^{13}$, P. H. Regan$^{13}$, C. Sigl$^{13}$, H. Schaffner$^{13}$, C. Scheidenberger$^{13}$, S. Schwertel$^{13}$, P.-A. Söderström$^{13}$, S. J. Steer$^{13}$, A. Stolz$^{13}$ & P. Strömmen$^{13}$


Coulomb Excitation of $^{104}$Sn and the Strength of the $^{100}$Sn Shell Closure

G. Guastalla$^1$, D. D. DiJulio$^2$, M. Görski$^3$, J. Cederkäll$^2$, P. Bountachkov$^{1,2}$, P. Golubev$^2$, S. Pietri$^1$, H. Graebe$^2$, F. Nowacki$^1$, K. Steigl$^1$, A. Alber$^6$, F. Amel$^7$, T. Arici$^2$, A. Atac$^2$, M. A. Bentley$^1$, A. Blazhev$^{10}$, D. Bloom$^9$, S. Brambilla$^{11}$, N. Braun$^{10}$, F. Camera$^1$, Zs. Dombrádi$^2$, C. Domingo-Pardo$^2$, A. Estrade$^2$, F. Farinon$^2$, J. Ger$^1$, N. Goel$^{1,3}$, I. J. Grégoire$^{13}$, T. Habermann$^{3,13}$, R. Hölschen$^2$, K. Jansson$^3$, J. Jolie$^{10}$, A. Jungclaus$^1$, I. Kojouharov$^2$, R. Knebel$^{13}$, R. Kumor$^{15}$, J. Kurcewicz$^{16}$, N. Kurz$^6$, N. Lalović$^2$, E. Merchan$^{1,3}$, K. Moschner$^{13}$, F. Naqi$^{3,10}$, B. S. Nara Singh$^9$, J. Nyberg$^{17}$, C. Nociforo$^3$, A. Oberfell$^{18}$, M. Pfützner$^{3,16}$, N. Pietralla$^1$, Zs. Podolyák$^{19}$, A. Procházková$^3$, D. Räler$^{1,13}$, P. Reiter$^{10}$, D. Rudolph$^2$, H. Schaffner$^2$, F. Schirru$^{19}$, L. Scurtu$^6$, D. Sohler$^2$, T. Swaleh$^2$, J. Taponige$^{10,22}$, Z. Vajta$^9$, R. Wadsworth$^9$, N. Warr$^{10}$, H. Weck$^3$, A. Wendt$^{10}$, O. Wieland$^{11}$, J. S. Winfield$^{2}$, and H. J. Wollersheim$^{9}$


Transition probabilities near $^{100}$Sn and the stability of the $N, Z = 50$ shell closure

T. Bäck$^{1,4}$, C. Qi$^1$, B. Cederwall$^1$, R. Liotta$^1$, F. Ghazi Moradi$^1$, A. Johnson$^1$, R. Wyss$^1$, and R. Wadsworth$^2$


Lifetime measurement of the first excited 2$^+$ state in $^{100}$Te

T. Bäck$^{1,4}$, C. Qi$^1$, F. Ghazi Moradi$^1$, B. Cederwall$^1$, A. Johnson$^1$, R. Liotta$^1$, R. Wyss$^1$, H. Al-Aziz$^2$, D. Blox$^1$, T. Brock$^2$, R. Wadsworth$^2$, T. Grzanka$^3$, P. T. Greenlees$^1$, K. Hauschild$^{1,3}$, A. Herranz$^{1,5}$, U. Jentsch$^{1,3}$, P. Jones$^1$, R. Juhls$^{1,3}$, S. Kutuzov$^3$, S. Ketelhut$^3$, M. Leino$^1$, A. Lopez-Martens$^{1,4}$, P. Nieminen$^1$, P. Peura$^1$, P. Rahikka$^1$, S. Rinta-Antila$^3$, P. Ruotsalainen$^2$, M. Sandzelius$^1$, J. Sara$^1$, C. Schoubye$^1$, I. Søren$^1$, J. Ustioleto$^1$, S. Go$^1$, E. Iseguchi$^1$, D. M. Callen$^1$, M. G. Procter$^1$, T. Braunroth$^1$, A. Dewald$^1$, C. Fransen$^1$, M. Hackstein$^1$, J. Litzinger$^1$, and W. Rothe$^1$
Spectra of the heaviest even-even $N=Z$ nuclei $^{88}\text{Ru}$ and $^{92}\text{Pd}$ were reported in 2001 and 2011, respectively. N. Mărginean et al., PRC 63, 031303(R) (2001); B. Cederwall et al., Nature 469, 68 (2011).

How to understand the equidistant pattern of the $^{92}\text{Pd}$ spectrum?
Averaged number of particles in different shells for $^{92}$Pd

What is the ’minimal’ model space one needs?

No-core shell model for $^4$He
M.D. Schuster et al., arXiv:1304.5491
The striking feature is that if we project it on to np coupled terms, the wave function can be represented by a single term $(\nu\pi)_9 \otimes (\nu\pi)_9$

$$\langle [j_p j_n(J_1) j_p j_n(J_2)] | [j_p^2(j_p) j_n^2(j_n)] , J \rangle = -2 \hat{J}_1 \hat{J}_2 \hat{J}_p \hat{J}_n \left\{ \begin{array}{ccc} j & j & J_p \\ j & j & J_n \\ J_1 & J_2 & J \end{array} \right\}$$

$^{96}$Cd (2n-2p): A simple example to show the pair content

Usually the wave function can be expanded as

$$|\Psi_I\rangle = \sum_{J_p, J_n} X_I(J_p J_n) |j_\pi^2(J_p) j_\nu^2(J_n); I\rangle,$$

The thus obtained wave function is a mixture of many component as a result of the np interaction.

$|\Psi_0(gs)\rangle = 0.76[|\pi^2(0)\nu^2(0)|I\rangle + 0.57[|\pi^2(2)\nu^2(2)|I\rangle + 0.24[|\pi^2(4)\nu^2(4)|I\rangle + 0.13[|\pi^2(6)\nu^2(6)|I\rangle + 0.14[|\pi^2(8)\nu^2(8)|I\rangle.$

CQ et al, PRC 84, 021301(R) (2011).
$^{96}$Cd (2n-2p): A simple example to show the pair content

The calculated spectrum show a equidistant pattern along the yrast line up to $I=6$; A naive picture is that the angular momenta of the states are generated by the rearrangement of the angular momentum vectors of the aligned np pairs.
Average number of pairs

\[
\langle \Psi_N \| (a_i^\dagger a_j^\dagger)_{J^\pi} \times (a_i a_j)_{J^\pi} \| \Psi_N \rangle
\]

\[
E_I = C_J^I V_J.
\]

\[
C_J^I (nn) = C_J^I (pp) = C_J^I (np)
\]

CQ, PRC 81, 034318 (2010)
Aligned np pair to explain the rotational-like spectra in $^{20}\text{Ne}$ and $^{44}\text{Ti}$

The spin-aligned pair plays a crucial role

It is strongly attractive since this maximally aligned configuration has maximal overlap between the proton and neutron wave functions

**Competition between the np aligned coupling and like nucleon aligned coupling?**

\[ V_9 = \langle (g_{9/2})_{J=9} | V | (g_{9/2})_{J=9} \rangle, \quad V_9(\delta) = V_9(1 + \delta) \]
Two particle coefficients of fractional parentage

$$\left\{ \left( j^2 \right)_{J_1} \left( j^2 \right)_{J_2} \left( j^2 \right)_{J_3} \left( j^2 \right)_{J_4} \left( j^2 \right)_{J_5} \left( j^2 \right)_{J_6} \right\}_I$$

CQ et al, PRC 84, 021301(R) (2011).
The relative motion of the np pairs

The transition strengths remain approximately the same along the yrast line.

The $B(E2)$ values connecting yrast states in $^{92}\text{Pd}$ are two times larger than those of $^{96}\text{Cd}$. 
Quartet-like coupling

The four $J = 9$ $np$ pairs in $^{92}$Pd can couple in various ways. With the help of two-particle cfp one may express the wave function in terms of $(((\nu \pi)_{9} \otimes (\nu \pi)_{9})_{I}, \otimes (\nu \pi)_{9})_{I''} \otimes (\nu \pi)_{9})_{I}$. 

Table I. Configurations with the largest probabilities for the state $^{92}$Pd$(0^{-})$ corresponding to the tensorial products of different two-particle states (upper) and four-particle states (lower).

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$x^2$</th>
</tr>
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<tbody>
<tr>
<td>$</td>
<td>\gamma_2 = 9^{+}\gamma'_2 = 9^{+}\gamma''_2 = 9^{+}\gamma'''_2 = 9^{+}\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>\gamma_2 = 9^{+}\gamma'_2 = 9^{+}\alpha_2 = 0^{+}\beta_2 = 0^{+}\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>\gamma_2 = 8^{+}\gamma'_2 = 1^{+}\alpha_2 = 0^{+}\beta_2 = 8^{+}\rangle$</td>
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<tr>
<td>$</td>
<td>\gamma_4 = 0^{+}_{1}\gamma'<em>4 = 0^{+}</em>{1}\rangle$</td>
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<td>$</td>
<td>\gamma_4 = 8^{+}_{1}\gamma'<em>4 = 8^{+}</em>{1}\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>\gamma_4 = 8^{+}_{2}\gamma'<em>4 = 8^{+}</em>{2}\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>\gamma_4 = 16^{+}_{1}\gamma'<em>4 = 16^{+}</em>{1}\rangle$</td>
</tr>
</tbody>
</table>

0$_{9/2}^{-}$-shell description of $^{31}$Ag (3p-3n): $7^{-}$

For simplicity, the Hamiltonian only contain the two matrix elements $V_0$ and $V_0$. The wave function is dominated by the configuration of $|((\Omega^2)_{9}((\Omega^2)_{9}|\otimes (\Omega^2)_{9})_{I}$. Calculation with a realistic Hamiltonian gives a even larger value.
Calculations in different spaces for $^{94}$Ag (3p-3n)

The ground state spin is calculated to be $0^+$. The lowest $T = 0$ state is $7^+$. The wave function is dominated by $\left| [(j^2)^9(j^2)_9]_{16}(j^2)_9 \right>_I=7$.

A stretch isomer?

$^{94}$Ag

<table>
<thead>
<tr>
<th>0$^+_f$</th>
<th>0$^+_p$</th>
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</table>

$^{94}$Ag
Interacting Boson models with aligned np pair

Table IV. Overlaps of the \((1g_{9/2})^4\) yrast eigenstates of the SLGT0 interaction with angular momentum \(J\) and isospin \(T = 0\) with various two-pair states, expressed in percentages.

<table>
<thead>
<tr>
<th>(J)</th>
<th>(B^2)</th>
<th>(S_P)</th>
<th>(D^2)</th>
<th>(D_G)</th>
<th>(D_I)</th>
<th>(D_K)</th>
<th>(G^2)</th>
<th>(I^2)</th>
<th>(K^2)</th>
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<td>0</td>
<td>91</td>
<td>80</td>
<td>35</td>
<td>18</td>
<td>7.4</td>
<td>1.9</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>97</td>
<td>85</td>
<td>17</td>
<td>22</td>
<td>1.5</td>
<td>0.0</td>
<td>0.4</td>
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<td></td>
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<tr>
<td>4</td>
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<td>11</td>
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<td>0.2</td>
<td>0.0</td>
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</tr>
<tr>
<td>6</td>
<td>55</td>
<td>70</td>
<td>43</td>
<td>0.2</td>
<td>4.3</td>
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</tr>
<tr>
<td>8</td>
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</tr>
<tr>
<td>10</td>
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<td>6.1</td>
<td>0.5</td>
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<tr>
<td>12</td>
<td>88</td>
<td>57</td>
<td>1.5</td>
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</table>


Pair truncation shell model approaches

Different facets of the nucleus

The np aligned pair wave functions can be a ‘good’ eigen state of the QQ interaction.
Summary

• Systems with particles in a single-j shell
• The np pair correlation from the binding energy difference
• Neutron-proton spin-aligned pair coupling in N=Z nuclei
• Ongoing:
  Extension to systems with many shells

The response to deformation effects
Application in large scale computing: Computer favors uncoupled scheme
Thank you!