The appearance of discontinuities in MHD

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Contents

- Motivation: pulsar scintillations
- Topological considerations
- Simulations
Pulsar scintillations

- Observed with essentially all pulsars
- First reference: Scheuer 1968
- Compared to twinkling of stars:
  - also caused by varying refractive index
  - but discrete structures rather than continuous variations
Stinebring et al. 2001: „parabolic structures in the Fourier-transformed dynamical spectra of strongly scintillating pulsars“

Dynamic spectrum (top) and its secondary spectrum
Flux density - linear grey-scale; black indicating highest flux
Grey-scale is logarithmic (linear in decibels) with a 48 dB range
How to produce scintillations

- Refractive index depends on electron density
- Diffraction/refraction by clumps (clumps within sheets?) or inhomogenous sheets
- Consensus that structures are sheets nearly aligned to line-of-sight
Nature of structures

- Scattering over angles 1-100 mas
- Must be almost aligned to line-of-sight: sheets more likely than filaments
- Sheets ~0.1 A.U. thick?
- Intervals of ~0.1 pc between sheets?
- Longevity uncertain
- Density contrast badly constrained, could be of order unity
Figure 2. The gradient image of linear polarization, $|\nabla P|$, for an 18-deg$^2$ region of the Southern Galactic Plane Survey. $|\nabla P|$ has been derived by applying Equation [1] to the $Q$ and $U$ images from Fig. 1; note that $|\nabla P|$ cannot be constructed from the scalar quantity $P \equiv (Q^2 + U^2)^{1/2}$, but is derived from the vector field $P \equiv (Q, U)$. $|\nabla P|$ is a gradient in one dimension, for which the appropriate units are (beam)$^{-0.5}$. Because $P$ measures linearly polarized intensity in units of millijanskys per beam, $|\nabla P|$ has units of millijanskys per (beam)$^{1.5}$. The scale showing $|\nabla P|$ is shown to the right of the image, and ranges from 0 to 15 mJy per (beam)$^{1.5}$. The inset shows an expanded version of the structure with highest $|\nabla P|$, covering a box of side 0:9 centred on Galactic longitude 329$^\circ$.8 and Galactic latitude +1$^\circ$.0. Plotted in the inset is the direction of $\nabla P$ at each position, defined as \[
arg(\nabla P) \equiv \tan^{-1} \left[ \text{sign} \left( \frac{\partial Q}{\partial x} \frac{\partial Q}{\partial y} + \frac{\partial U}{\partial x} \frac{\partial U}{\partial y} \right) \sqrt{\left( \frac{\partial Q}{\partial y} \right)^2 + \left( \frac{\partial U}{\partial y} \right)^2 / \sqrt{\left( \frac{\partial Q}{\partial x} \right)^2 + \left( \frac{\partial U}{\partial x} \right)^2}} \right]. \]
For clarity, vectors are only shown at points where the amplitude of the gradient is greater than 5 mJy per (beam)$^{1.5}$. 

Gaensler et al. 2011, 21cm continuum
Conditions in the ISM

- Turbulence driven by galactic differential rotation, stellar winds & UV, supernovae, etc.
- Locally, in absence of driving, turbulence decays: local dynamic timescale << external driving timescale
Equilibria and the momentum equation

- Momentum equation:
  \[
  \rho \frac{\text{d} \mathbf{u}}{\text{d} t} = -\nabla P + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}
  \]

  inertia                  pressure                Lorentz
  gradient                 force

- In fluid of infinite conductivity, field lines are ‘frozen’ into fluid

- \[
  \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})
  \]
Equilibria and the momentum equation

- Momentum equation:

\[
\rho \frac{d\mathbf{u}}{dt} = -\nabla P + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}
\]

  - Inertia
  - Pressure gradient
  - Lorentz force

- In fluid of infinite conductivity, field lines are ‘frozen’ into fluid
- Arbitrary field has *volume-filling* field lines
- Equilibria have a special topology; field lines lie in ‘magnetic surfaces’ of constant pressure. Equilibrium field lines are *area-filling*
- To reach equilibrium, flux-freezing must break down)
- How? Diffusion time is too long
- Current sheets expected (e.g. Arnold 1986 & Gruzinov 2009)
Formation of current sheets

Demonstrated numerically in 2D (Gruzinov 2009)

Fig. 2.— *Left*: Initial isolines of $\psi$. *Right*: Final isolines of $\psi$, also shown (thick colored lines) are the isolines of $\Delta \psi$.

Fig. 3.— Graphical "proof" that singular current layers form from the X-points

Fig. 4.— *Left*: Initial isolines of $\psi$. *Right*: Final isolines of $\psi$. Also shown (thick colored line) is the theoretical separatrix – two 251° circular arcs and a line segment.
Simulations of MHD relaxation

Aim: demonstrate formation of current sheets during MHD relaxation in 3-D

Code: Åke Nordlund’s STAGGER-CODE

Setup: cube, compressible, lowish diffusivity, \( \text{Pr}_m = 10 \)

Initial conditions: constant density and pressure, smoothly varying random magnetic field with plasma-\( \beta = 0.5 \)
What happens in the simulations?

- Motion on Alfvén timescale
- Current sheets form!
- Magnetic energy drops; helicity conserved
- Energy minimum is approached:
  \[ E \rightarrow k_{\text{min}} H \text{ where } k_{\text{min}} = \frac{2\pi}{L_{\text{box}}} \]
- Same behaviour at high and low plasma-\(\beta\)
Simulation

Discontinuities (current sheets) appear spontaneously, allowing reconnection, i.e. topological reorganisation.
Energy & helicity

Thick lines are energy
thin lines are helicity

three runs, same $E_{\text{init}}$, different $H_{\text{init}}$

Time unit $\sim$ Alfven time
Summary

 april sheets allow topological rearrangement on dynamic timescale

Should form spontaneously in the ISM, where turbulence is intermittent

Could be responsible for pulsar scintillations!

Should also form in stars(!)

Similar to coronal heating `problem` where current sheets form in response to motions at photosphere (e.g. Parker, various papers; Galsgaard & Nordlund 1996)
Formation of discontinuities via driving at the boundary

Fig. courtesy of E. Parker