Meridional Circulation in Solar and Stellar Convection Zones

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Differential Rotation and Magnetism Across the HR Diagram,
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1) Mean Flows Regimes
   - Fast and slow rotators

2) Maintenance of Meridional Circulation
   - Gyroscopic Pumping
   - Angular momentum transport by the Reynolds Stress

3) Application to the Sun and Stars
   - Is the Sun Fast or Slow?
Solar and Anti-Solar Differential Rotation

\[ \Omega \]

\[ R_o = \frac{U}{2\Omega L} \]

Gastine, Wicht & Aurnou (2013)

Simulations suggest two different regimes for the Mean Flows

**Fast Rotation (Ro ≤ 0.3)**

- Multi-celled MC (poleward flow in lower CZ near equator)
- Solar-like DR (fast equator)
- red CW blue CCW

**Slow Rotation (Ro ≥ 0.9)**

- Single-celled MC (equatorward flow in lower CZ)
- Anti-Solar DR (fast poles)

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**Flux-Transport dynamo models may have difficulty operating in rapidly-rotating stars**

Featherstone & Miesch (2013)
Regimes can be achieved by varying $\Omega$ or by varying dissipation $\nu$ and $\kappa$.

\[
\kappa \left. \frac{\partial S}{\partial r} \right|_{\text{top}} \propto L
\]

\[
R_o = \frac{U}{2\Omega L}
\]
Regimes can be achieved by varying $\Omega$ or by varying dissipation $\nu$ and $\kappa$.

$$R_o = \frac{U}{2\Omega L}$$
2) GP

Comes from the zonal component of the MHD momentum equation, averaged over longitude and time

\[ \frac{\partial}{\partial t} (\rho \mathcal{L}) + \langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F} \]

\[ \mathcal{F} = -\nabla \cdot [\lambda \langle \rho \mathbf{v}' \mathbf{v}'_\phi \rangle - \lambda \langle \mathbf{B} \mathbf{B}_\phi \rangle - \rho \nu \lambda^2 \nabla \Omega] \]

\[ \mathcal{L} = \lambda^2 \Omega = \lambda \langle v_\phi \rangle \]

Reynolds stress  
Lorentz force  
Viscous diffusion

No assumptions beyond basic MHD!

(GONG inversions, courtesy Rachel Howe)

\[ \nabla \mathcal{L} \approx \frac{d \mathcal{L}}{d \lambda} \hat{\lambda} \]

\[ \Omega/2\pi \]

\[ L = \lambda^2 \Omega \]

Prograde torque  \[ \mathcal{F} > 0 \]

Retrograde torque  \[ \mathcal{F} < 0 \]
Mean Flow Profiles Determined mainly by convective angular momentum transport

\[ \mathcal{F}_{RS} = -\nabla \cdot \mathbf{F}_{RS} \]

**Fast Rotation**
(Ro << 1)

**Slow Rotation**
(Ro >> 1)

Competition between

Radially inward transport at high latitudes
(*ballistic plumes*)

Cylindrically outward transport at low latitudes
(*banana cells*)

Convergence

Divergence

\[ \langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F} \]

Featherstone & Miesch (2013)
Mean Flow Profiles Determined mainly by convective angular momentum transport

\[ \nabla \cdot \mathbf{F}_{\text{ng}} = -\nabla \cdot \mathbf{F}_{\text{ng}} \]

Featherstone & Miesch (2013)

Fast Rotation (\( \text{Ro} \ll 1 \))

Slow Rotation (\( \text{Ro} \gg 1 \))

Convergence

Divergence

Gyroscopic pumping of the meridional flow

Equatorial plane

\[ \langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F} \]
Mean Flow Profiles Determined mainly by convective angular momentum transport

Fast Rotation (\(Ro \ll 1\))

\[ \mathbf{F}_{\text{RS}} = -\nabla \cdot \mathbf{F}_{\text{RS}} \]

Gyroscopic pumping of the meridional flow

Convergence

Divergence

\[ \langle \rho \mathbf{v}_m \rangle \cdot \nabla L = \mathcal{F} \]

Slow Rotation (\(Ro \gg 1\))

Equatorial plane

Featherstone & Miesch (2013)
Mean Flow Profiles Determined mainly by convective angular momentum transport

\[ F_{\text{res}} = - \nabla \cdot F_{\text{res}} \]

Fast Rotation 
(\( Ro \ll 1 \))

Convergence

Divergence

Gyroscopic pumping of the meridional flow

Equatorial plane

\[ \langle \rho v_m \rangle \cdot \nabla \mathcal{L} = F \]

Featherstone & Miesch (2013)
Latitude-Dependent Rossby Number

Consider Coriolis force acting on a Downward plume driven by surface cooling

\[
\frac{\partial v_r}{\partial t} = 2\Omega_0 \sin \theta \ v_\phi + \ldots
\]

\[
\frac{\partial v_\phi}{\partial t} = -2\Omega_0 \sin \theta \ v_r + \ldots
\]

\[
R_o = \frac{v_r}{2\Omega_0 D \sin \theta} = \frac{r_c}{D}
\]

\[
r_c = \left| \frac{v_r^2}{\partial v_\phi/\partial t} \right| = \frac{v_r}{2\Omega_0 \sin \theta}
\]

Transition between fast/slow regimes occurs as the latitude where Ro crosses 1 shifts equatorward. Effective Ro large at high latitudes, smaller at low latitudes.
The RS transition

The transition between fast & slow rotating regimes is regulated by (the divergent component of) the Reynolds stress

\[
F_{RS}(r, \theta) = \nabla \chi(r, \theta) + \nabla \times \left( \Gamma(r, \theta) \hat{\phi} \right)
\]

\[
\nabla^2 \chi = -\nabla \cdot F_{RS}
\]

Contours track increasing dominance of inward angular momentum transport at high latitudes

Also - indicate a shift from cylindrically outward to equatorward transport at low latitudes
It’s actually the MC that spins up the poles in the slow rotators.

Convective angular momentum transport establishes antisolar DR only indirectly, by inducing a meridional flow (GP) that transports angular momentum poleward.

**MC is essentially a runaway, axisymmetric convective instability** (equatorward $\nabla \langle S \rangle$)
So Where is the Sun?

The Sun \((R_\odot \lesssim 1)\) may be in an atypical state relative to other stars

('~ Faster Rotators (young stars: \(R_\odot \lesssim 0.3\))
- Cylindrical \(\Omega\) profile
- multi-celled MC

('~ Sun (Goldilocks: \(0.3 \lesssim R_\odot \lesssim 1\))
- Conical \(\Omega\) profile
- ?? MC

('~ Slower Rotators (old stars \(R_\odot \gtrsim 1\))
- Anti-solar \(\Omega\) profile
- single-celled MC

\[\frac{\partial \Omega^2}{\partial z} = \frac{g}{r \lambda C_P} \frac{\partial \langle S \rangle}{\partial \theta}\]

LHS scales as \(R_\odot^{-2}\)
RHS scales as \(R_\odot^{-1}\)

(TWB Breaks down for \(R_\odot > 1\))

Minimal \(\frac{\partial \Omega}{\partial r}\) at equator suggests there may be a convergence of the latitudinal angular momentum flux as seen in some slow rotators

⇒ Might produce a single-celled MC

(Hotta & Yokoyama 2011)
So Where is the Sun?

The Sun ($R_\odot \lesssim 1$) may be in an atypical state relative to other stars

♂ Faster Rotators (young stars: $R_\odot \lesssim 0.3$)
  - Cylindrical $\Omega$ profile
  - multi-celled MC

♂ Sun (Goldilocks: $0.3 \lesssim R_\odot \lesssim 1$)
  - Conical $\Omega$ profile
  - ?? MC

♂ Slower Rotators (old stars $R_\odot \gtrsim 1$)
  - Anti-solar $\Omega$ profile
  - single-celled MC

Minimal $\partial \Omega / \partial r$ at equator suggests there may be a convergence of the latitudinal angular momentum flux as seen in some slow rotators

⇒ Might produce a single-celled MC

\[
\frac{\partial \Omega^2}{\partial z} = \frac{g}{r \lambda C_P} \frac{\partial \langle S \rangle}{\partial \theta}
\]

LHS scales as $R_\odot^{-2}$

RHS scales as $R_\odot^{-1}

(Hotta & Yokoyama 2011)

(TWB Breaks down for $R_\odot > 1$)
Near the Transition

*Solar-like DR*

*but single-celled MC!*

\[-\nabla \cdot \mathbf{F}_{RS}\]
Turn the Problem around

Take solar \( \Omega \) profile and ask what RS would be needed to sustain a single-cell profile

\[
\langle \rho v_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F}
\]

\[
\Omega - \Omega_0 = \mathcal{L} = \lambda^2 \Omega
\]

Summary: Inconclusive!
The sun is close to the transition between single and multiple-celled MC profiles; could go either way

Miesch (2005)
Summary

Mean Flow regimes
- Fast rotators: solar-like DR, multi-celled MC
- Slow rotators: anti-solar DR, single-celled MC
- Sun may be near the transition

Angular momentum transport by the RS
- Ballistic plumes at high latitudes (radially inward)
- Banana cells at low latitudes (cylindrically outward)
- Latitude where effective Rossby number crosses $O(1)$ regulates fast/slow transition
- Polar spin-up in slow rotators attributed to MC

Maintenance of MC
- Zonal forcing (RS)
- Gyroscopic pumping
Mean flow regimes reflected in stratification (specific entropy $S$)

**residual entropy**

- **Fast Rotator**
  - $+/- 200$

- **Slow Rotator**
  - $+/- 400$

Polar vortex acts as a transport barrier for slow rotators

\[
\frac{\partial \Omega^2}{\partial z} = \frac{g}{r \lambda C_P} \frac{\partial \langle S \rangle}{\partial \theta}
\]

**equator**

- pole ($85^\circ$)

**mean**

**Fast Rotator**

**Slow Rotator**