Solar differential rotation hints to obtain a near-surface shear layer

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Outline

1. Large-scale flows in the solar interior
2. Mean-field models, DNS and LES
3. Anelastic simulations with EULAG
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   b) Convergence
   c) Near-surface shear layer
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1. Large-scale motions in the solar interior
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Ulrich (2010)
Dikpati et al. (2004)
Mitra-Kraev & Thompson (2007)

Cycle 23 average

Vmerid (m/s)

Pole

Tachocline

GONG

MDI

Howe et al., 2003

Courtesy J. Zhao

Mitra-Kraev & Thompson (2007)
2. mean-field models
DNS and LES

Large-scale motions as well as large-scale magnetic field are the result of collective effects of turbulence

\[
\frac{\partial \overline{B}}{\partial t} = \nabla \times \left( \overline{U} \times \overline{B} + E + \eta \nabla \times \overline{B} \right)
\]

\[\mathcal{E} = \langle \mathbf{u}' \times \mathbf{b}' \rangle\]  

\[
\frac{D \overline{U}}{Dt} = -\frac{\nabla p}{\rho} - \nabla \cdot (\rho Q) + \rho \mathbf{g} + \nabla \cdot \pi
\]

\[Q_{ij} = \langle u'_i u'_j \rangle\]  

(Steenbeck et al. 66)

(Kitchatinov & Rudiger 93)
(Kitchatinov & Rudiger 95)
Direct Numerical Simulations (DNS)

Resolve scales down to the grid resolution. They could be global or local.

\[ N^3 \geq Re^{9/4} \]

\[ Re \approx 10^{12} \ldots 10^{13} \]

(Käpylä et al. 2011)
Large eddy Simulations (LES)

Resolve scales down to the grid resolution, the sub-grid scale (SGS) contribution is considered via a turbulent model.

\[ \nu_{SGS} = \text{cnt} \gg \nu \]

\[ \nu_{SGS} \propto \rho^{-1}(\rho^{-1/2}) \gg \nu \]

(Gilman 1976)

- ASH code, e.g. Miesch et al. (2011)
  - Anelastic approximations, LES.
Implicit sub-grid scale modeling (ISGS, ILES): the non-linear computation of the truncation error (i.e., numerical viscosity) is identified with the SGS contribution (EULAG code), (Margolin & Rider, 2002)

\[ \nu_{SGS} = C_s \Delta \left( \pi_{ij} \pi_{ij} \right)^2 \]

\[ C_s = \text{ctn (Smagorinsky)} \]

\[ C_s = C_s(U) \text{ (Dynamic Smagorinsky)} \]
Ghizeru, Charbonneau & Smolarkiewicz (2010)
3. Anelastic simulations with EULAG code

Anelastic approximation
(Lipps & Helmer 1982, Lipps 1990)

\[ \nabla \cdot (\rho_0 \mathbf{u}) = 0, \quad (1) \]

\[ \frac{D \mathbf{u}}{Dt} - 2\Omega \times \mathbf{u} = -\nabla \left( \frac{p'}{\rho_0} \right) + g \frac{\Theta'}{\Theta_0} + \frac{F}{\rho_0}, \quad (2) \]

\[ \frac{D \Theta'}{Dt} = -\mathbf{u} \cdot \nabla \Theta_e + \frac{1}{\rho_0} \mathcal{H}(\Theta') - \alpha \Theta' \quad (3) \]

\[ ds = c_p d \ln \Theta \quad (4) \]
The background state, $\rho_0$, $\rho_0$, $\Theta_0$

Ambient state, is a horizontal and time average of a “known” solution, e.g., solar model S, polytropic model.
a) Angular momentum transport
Rotation vs. Convection

Rotational regimes due to convection in rotating spherical shell

- Solid rotation
- High latitude acceleration
- Equatorial acceleration

Rayleigh number $R \approx 7.8T^{2/3}$

Rayleigh number $R \approx 0.84T^{2/3}$

Prandtl Number $P=1$
Stress free boundaries
Angular momentum conservation

\[
\frac{\partial (r \phi \rho \mathbf{u})}{\partial t} = \frac{1}{r \sin \theta} \nabla \cdot \left( \rho r \sin \theta \left[ -\nu \nabla \mathbf{u}_\phi + (\mathbf{u}_\phi + \Omega_0 r \sin \theta) \mathbf{u} + \mathbf{u}_\phi' \mathbf{u}' \right] \right)
\]

- Viscous flux: \( F_{vis} = -\nu \nabla \mathbf{u}_\phi \)
- Meridional circulation: \( F_{MC} = (\mathbf{u}_\phi + \Omega_0 r \sin \theta) \mathbf{u} \)
- Reynold stresses flux: \( F_{RS} = \mathbf{u}_\phi' \mathbf{u}' \)

\[
\begin{align*}
(\lambda, \phi, z) \\
\lambda &= r \sin \theta \\
z &= r \cos \theta
\end{align*}
\]

\[
\begin{align*}
F_\lambda \hat{\lambda} &= F_r \sin \theta \hat{r} + F_\theta \cos \theta \hat{\theta} \\
F_z \hat{z} &= F_r \cos \theta \hat{r} - F_\theta \sin \theta \hat{\theta} \\
Q_{\phi r}, Q_{\phi \theta}, Q_{r \theta}
\end{align*}
\]
a) $\Omega_0 = 112$ days ($\frac{1}{4}\Omega_\odot$, $Re \sim 60$)

b) $\Omega_0 = 56$ days ($\frac{1}{2}\Omega_\odot$, $Re \sim 60$)
c) $\Omega_0 = 28\text{days} \ (\Omega_\odot, \ Re \sim 60)$

d) $\Omega_0 = 14\text{days} \ (2\Omega_\odot, \ Re \sim 60)$
Gastine et al. (2012)
b) Convergence: \( x=128x64x47 \)
c) Near-surface shear layer

Based in models slow rotating models where convection dominates over rotation …

Negative radial shear appears at lower latitudes
CONCLUSIONS

• EULAG code is a powerful tool to study solar/stellar rotation and dynamo, high Re are obtained for relatively low resolutions.

• The resulting rotation profile depends on the balance between Coriolis and buoyancy forces. For the stratification considered here solar-like rotation (fast equator) is obtained at the solar rotation rate.

• Due to the highly sub-adiabatic stable region, all the simulations exhibit the formation of a tachocline.

• Taylor-Proudman columnar rotation profiles are persistent, indicating deficient latitudinal turbulent heat transfer.
• For all the simulations at the solar rotation rate the meridional flow is multicellular

• Vigorous convection at the upper part of the convection zone leads to the formation of a “near-surface shear layer” at equatorial latitudes