Rotational Effects on Convection

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Conference on Differential Rotation and Magnetism across the HR Diagram
Two Big Puzzles: Jets and Spots
Mean Zonal Winds

Jupiter

Saturn

Uranus

Neptune
Long-lived Vortices


• Mantere et al. 2011, *Dependence of the large-scale vortex instability on latitude*, stratification, and domain size, Astron. Nachr., 332: 876-882

3D LES of Turbulent Compressible Convection in a f-Box

\[
\begin{align*}
\frac{\partial}{\partial t} \rho &= -\nabla \cdot (\rho v) \\
\frac{\partial}{\partial t} (\rho v) &= -\nabla \cdot (\rho vv) - \nabla p + \nabla \cdot \Sigma + \rho g + 2\rho v \times \Omega \\
\frac{\partial}{\partial t} E &= -\nabla \cdot \{(E + p)v - v \cdot \Sigma + f\} + \rho v \cdot g
\end{align*}
\]
Model Description

- Constant angular velocity vector
- Depth ~ 4 pressure scale heights; aspect ratio = 4; $\gamma = 5/3$
- Heat flux = 0.03125 pressure x $C_s$; $C_s =$ isothermal sound speed at top
- Units: length = $d$, time = $d/C_s$, thermal quantities = values at top
- Latitude = 67.5, 22.5; $\Omega = 0.375, 0.75, 1.5, 3.$
- Conservative finite-difference scheme; 540x540x80 grids
- $Re \sim 2 \times 10^3 - 10^4$; $Ta \sim 2 \times 10^8 - 10^{10}$

Three ways to define the Coriolis number:

\[
Co1 = f_0 \frac{d}{V} \quad V = \text{rms velocity}, \quad f_0 = 2\Omega
\]

\[
Co2 = f_0 \frac{d}{V_0} \quad V_0 = V \text{ of the non-rotating case}
\]

\[
Co3 = f \frac{d}{V_0} \quad f = 2\Omega \cos (\text{colatitude})
\]
# Case Set 1

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda$</th>
<th>$\Omega$</th>
<th>$V$</th>
<th>$\text{Co1 } f_0d/V$</th>
<th>$\text{Co2 } f_0d/V_0$</th>
<th>$\text{Co3 } f d/V_0$</th>
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<td>0.285</td>
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<td>34.5</td>
<td>13.2</td>
</tr>
</tbody>
</table>
Regime Diagram

\[ \text{flux} = 0.03125 \]

- **Cyclones**
- **Anticyclones**
- **Cellular**

Threshold to cyclones: Co3
Threshold to anticyclones: Co3
Vertical Cuts of Velocity Fields through Centers of Cyclone and Anticyclone

\( \Omega = 3, \text{lat} = 67.5^\circ \)

**vertical velocities**

- **cyclone**
- **anticyclone**
Regime Diagram (more cases)
f = 0.0375/8, Ω = 1.5; aspect ratio = 4 & 8
f = 0.0375/8, Ω = 3; aspect ratio = 8
Zonal Wind Bands

2D turbulence models

- Yano et al. 2005
  Geo. & Astro. Fluid Dyn. 99, 137-150

3D convection models

- Heimpel et al. 2007
  Icarus, 187, 540-557

- Chan et al. 2008
  JGR Planet
  doi: 10.1029/2008
Multiple jets with prograde equatorial wind

Horizontal cut of temperature
Broad Retrograde Equatorial Wind

Horizontal cut of temperature
Summary

• Long-lived cyclones and anticyclones can be generated in a rotating convection zone
• As Co increases, cyclones appear first
• Multiple zonal wind bands can be generated in a rotating convection shell
• Prograde (or retrograde) equatorial wind depends on Co.
• Long-lived vortices can be generated in co-existence with the zonal wind bands
Thank You!