The solar differential rotation in the 18th century

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Data base

- 1000 solar observations by Johann Caspar Staudacher in 1749-1799
- We have spot distributions, dates and times
- Can we measure the differential rotation?
Data base

- Ambiguous orientations of the solar disk
- Observations are only 17h apart
Data base

- Orientation of drawings is unknown
- Pairs of drawings are used to find
  - The orientations
  - The spot positions
  - The differential rotation parameter
Data base

- Another example
- It's not always obvious...
Analysis method

- We know what to expect:
  \[ \Omega = \Omega_{eq} + \delta\Omega \sin^2 b \]
  perhaps plus higher term

- We also know that sunspots are fairly stationary over time

- Good knowledge of the model is a perfect case of using Bayesian inference
Bayesian inference

- Do not touch the data!
- Compute what would have been observed, given your model and compare with data:

\[
\Lambda(x_j, y_j; \lambda_j, b_j, \delta \Omega, \Delta t) = \\
\prod_{j=1}^{n_i} \frac{1}{\sqrt{2\pi \sigma}} \exp \left[ - \left( \frac{[x_j - f_x(\lambda_j, b_j, \delta \Omega, \Delta t)]^2}{2\sigma^2} + \frac{[y_j - f_y(\lambda_j, b_j, \delta \Omega, \Delta t)]^2}{2\sigma^2} \right) \right],
\]
Bayesian inference

- Explore then entire parameter space for the probability to have produced the data
- Do this with Monte Carlo Markov chains

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<th>Spots</th>
<th>Measurements</th>
<th>Parameters</th>
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Results

- Full probability distribution for all combinations of parameters
- Integrated distribution gives “marginal distribution” for desired parameter
- Sun: \(-0.0501 \text{ /d}\) (Balthasar et al. 1986)
Results

- Split the period into two and see what happens
Differential rotation in dynamo

- Solve mean-field eq. for $B$ and $\Omega(r,\theta)$
- Include back-reaction on generation of diff. rotation
- Get maximum diff. rot after grand minimum

Küker, Arlt, Rüdiger (1999)
Summary

- Solar observations of 250 years ago can reproduce the differential rotation of the Sun
- Slight tendency of higher differential rotation just after the Maunder minimum
- Note the enormous potential in historical observations