1. **Exercice : Neutron stars and the EOS of a free Fermi gas of neutrons**

   — Calculate the pressure p and the energy density ε as a function of the baryon number density n_B. Save the results in a file for increasing baryon number density

   $$n_B(\mathrm{fm}^{-3}) \quad \varepsilon(\mathrm{MeV/fm}^3) \quad p(\mathrm{MeV/fm}^3)$$

   — Solve the TOV system

   $$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \varepsilon \\ \frac{d\Phi}{dr} &= \left(1 - \frac{2Gm}{rc^2}\right)^{-1} \left(\frac{Gm}{r^2} + 4\pi G \frac{p}{c^2} r\right) \\ \frac{dp}{dr} &= -\left(\varepsilon + \frac{p}{c^2}\right) \frac{d\Phi}{dr} \end{aligned}$$

   with this equation of state and give the gravitational mass as a function of the radius. Does (as often stated) the degeneracy pressure of the neutrons give the counterpart to gravitation ? What else could play a role ?

2. **Correction**

   — Here we assume a pure neutron gas, thus we do not have to worry about charge neutrality. β-equilibrium is of course not fulilled and our stars would be unstable with respect to weak neutron decay. But for the sake of the exercice, we suppose neutrons not to decay. The neutron chemical potential is equal to the baryon one, $\mu_n = \mu_B$ and the baryon number density is given by the neutron density,

   $$n_B = n_n = \frac{p_F^3}{3\pi^2(\hbar c)^3} \ , \tag{1}$$

   with $p_F = \sqrt{\mu_n^2 - m_n^2}$, the Fermi momentum.

   — The pressure and the energy density can be evaluated from the equations given in the lecture

   $$\begin{aligned} \varepsilon &= \frac{1}{8\pi^2(\hbar c)^3} \left(\mu_n p_F (2\mu_n^2 - m_n^2) - m_n^4 \log\left(\frac{\mu_n + p_F}{m_n}\right)\right) \\ p &= -\varepsilon + \mu_n n_n \end{aligned}$$

   — Boundary conditions for soving TOV equations are vanishing pressure at the surface and chosen conditions at the center (e.g. central density fomr which central pressure and energy density can be calculated). m is vanishing at the center. Note that Phi can be eliminated from the system.

   — The results are available on the web page in the file `neutrongas.dat` for the EoS and `neutrongas.out` for the outcome of TOV system, parameterised by the central density.

   — Conclusion : it is obvious that the maximum mass obtained with this EoS is well below the oserved neutron star masses, thus our simple model for the EoS is not viable and nuclear interaction has to be considered.

   — Useful constants : $m_n = 939.5 MeV$, $\hbar c = 197.33$ MeV fm, $G = 1.3235 \times 10^{-12}$ fm^3/MeV m^2, $M_\odot = 1.1156 \times 10^{15}$ MeV m^3/fm^3

— If you are interested, you can download a nuclear EoS from Compose, e.g. `https://compose.obspm.fr/eos/96/`, and solve again the TOV system. You only need to download the `eos.thermo` and the `eos.nb` file. The latter contains the grid points in baryon number density n_B in fm^{-3} and the former in column 2 the index for the grid point in n_B and in column 4 the pressure divided by n_B. The energy density ε is contained in column 10 in the form $\varepsilon/(m_n n_b) - 1$ (dimensionless). Solve again the TOV system to obtain mass and radius of a spherical star. You might also use these two files directly within the `Lorene` software, `https://lorene.obspm.fr` to obtain rotating neutron star models.