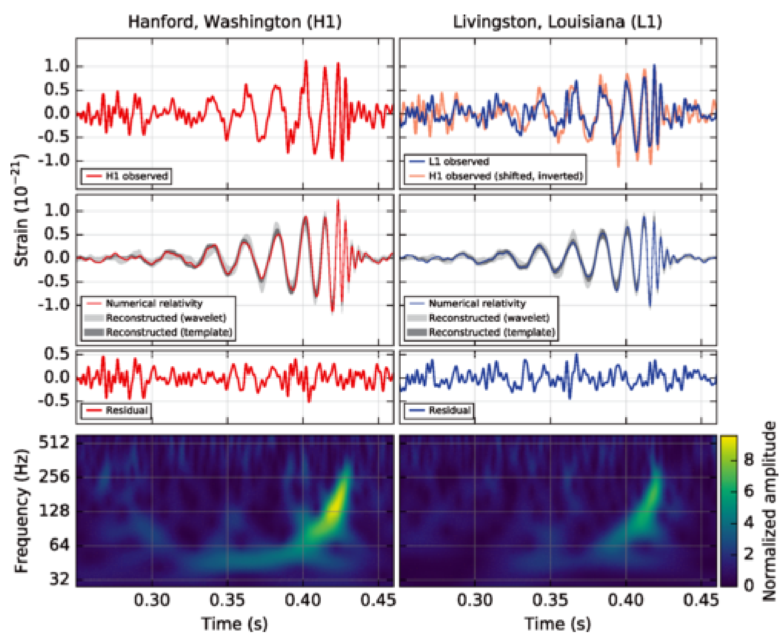
Foundations of Numerical Relativity

Course Outline

Numerical relativity has seen dramatic progress in the last decade. Numerical simulations of binary black holes, for example, have almost become routine, and have provided theoretical predictions for the gravitational waveforms that were directly detected for the first time two years ago (see below). Simultaneously, the field has advanced to model more complicated objects, including neutron stars and accretion disks in the presence of magnetic fields, and their processes, including tidal disruption and jet formation. Even after taking a class on general relativity (GR), however, it is probably difficult to follow the literature in numerical relativity. This short course is intended as a "gentle" introduction to the field. We will not assume any knowledge of (GR), and will introduce some of its most important objects and concepts. We will then discuss how Einstein's field equations of GR can be cast in a form that is suitable for numerical solution. We will introduce the notion of a "3+1" split between time and space in the context of electromagnetism, and will motivate how such a split results in a set of constraint equations and a set of evolution equations. Finally, we will discuss how the constraint equations can be solved to find so-called initial data, and how these initial data can then be integrated forward in time with the help of the evolution equations. This process results in computer simulations of black holes and neutron stars, and allows us to predict the gravitational wave emitted by such exotic objects.



B. P. Abbott *et.al.* (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. **116**, 061102 (2016)

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A tentative outline

- A brief review of GR (roughly Lecture 1)
  - Newton's gravity from a relativistic view
  - The spacetime metric
  - Einstein's field equations
  - Black holes
  - Gravitational Waves

- The 3+1 decomposition (roughly Lecture 2)
  - Scalar waves
  - Electromagnetism (E&M)
  - Foliations and spatial slices
  - The spatial metric
  - Constraint and evolution equations: the ADM equations
- Solving the constraint and evolution equations (roughly Lecture 3)
  - Conformal Transformations
  - Elementary solutions to the Hamiltonian constraint
  - Evolution equations in E&M and GR
  - Choosing the lapse and shift
  - Black hole simulations

Some References

Any textbook on General Relativity will be useful for a review of GR, but I particularly recommend

- T. Moore, *A General Relativity Workbook*, University Science Books
- S. Carroll, *Spacetime and Geometry*, Addison-Wesley

My lectures will mostly be based on

- T. W. Baumgarte & S. L. Shapiro, *Numerical Relativity: Solving Einsteins Equations on the Computer*, Cambridge University Press

but I also recommend

- M. Alcubierre, *Introduction to 3+1 Numerical Relativity*, Oxford University Press
- E. Gourgoulhon, *3+1 Formalism in General Relativity*, Springer
- C. Bona, C. Palenzuela-Luque & C. Bona-Casas, *Elements of Numerical Relativity and Relativistic Hydrodynamics*, Springer

Some Conventions

- We will use *geometrized units*, in which $c = G = 1$
- Indices $a, b, c, \ldots, h$ and $o, p, q, \ldots$ run over spacetime indices, while $i, j, k, \ldots, n$ run over spatial indices only ("Fortran" convention)
- We use the Einstein summation convention, by which we sum over repeated indices
- The flat spacetime (or space) metric is denoted by η_{ab} (or η_{ij}) in *any* coordinate system. Only in Cartesian (inertial) coordinates do we have $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$.