Foundations of Numerical Relativity

Exercises # 3

3.1 Consider *geodesic slicing* with

$$\alpha = 1, \qquad \beta^i = 0. \tag{1}$$

(a) Use the results of Problem 2.4 to show that $\partial_t K \geq 0$.
(b) It can be shown that $K_{ij}K^{ij} > K^2/3$. Use this lower bound as an approximation for $K_{ij}K^{ij}$ to find a differential equation for K alone.
(c) Solve the differential equation of part (b), assuming that $K = K_0 > 0$ at some time $t = t_0$. Find the time at which a coordinate singularity will develop, as $K \to \infty$. Express your answer in terms of K_0 and t_0.

3.2 Consider *harmonic slicing* with

$$\partial_t \alpha = -\alpha^2 K, \qquad \beta^i = 0. \tag{2}$$

(a) Use the results of Problem 2.4 to show that

$$\alpha = C(x^i)\,\gamma^{1/2}, \tag{3}$$

where $C(x^i)$ is a constant of integration that depends on the spatial coordinates x^i only.
(b) Now consider *1+log slicing* with

$$\partial_t \alpha = -2\alpha K \tag{4}$$

to show that $\alpha = 1 + \ln\gamma$ if we also choose $\beta^i = 0$. Here a constant of integration has been chosen appropriately. This result explains the name of this slicing condition.

3.3 Consider the reformulation of Maxwell's equations that we discussed in class,

$$\begin{aligned} \partial_t A_i &= -E_i - D_i\Phi \\ \partial_t E_i &= -D_j D^j A_i + D_i\Gamma - 4\pi j_i \\ \partial_t \Gamma &= -D_i D^i \Phi - 4\pi\rho. \end{aligned} \tag{5}$$

Show that, for this version of Maxwell's equations, the constraint violations $\mathcal{C} \equiv D_i E^i - 4\pi\rho$ satisfy the wave equation

$$(-\partial_t^2 + D_i D^i)\,\mathcal{C} = 0. \tag{6}$$

Assume that charge is conserved, so that the continuity equation $\partial_t \rho = -D_i j^i$ holds, and that space is flat, so that the covariant derivatives commute, $D_i D_j = D_j D_i$.