Yukawa Unification, WMAP Dark Matter and Observation of Charged Lepton Flavour Violation

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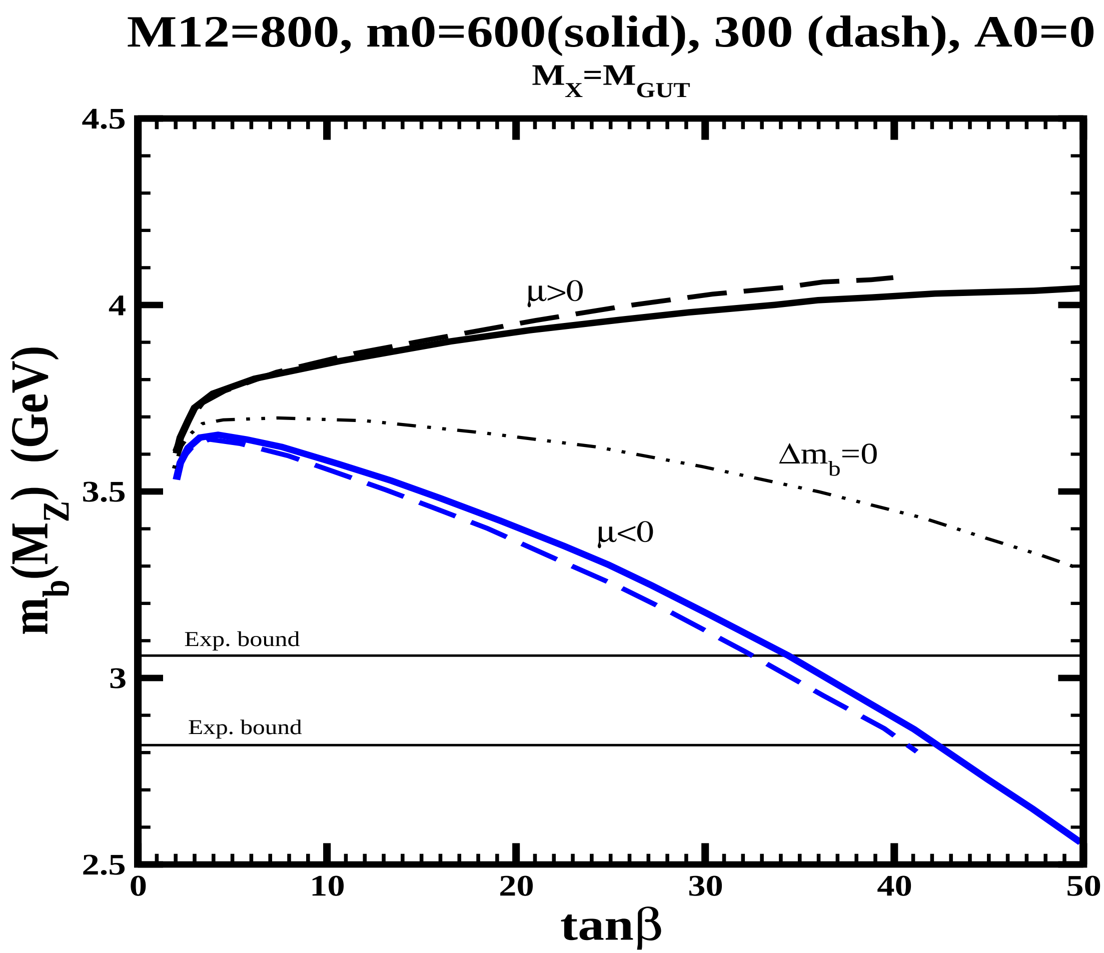
Introduction

- $SU(5)$ unification in SUSY and the m_b prediction.
- "*See-saw*" Neutrinos:
  - Size of the neutrino mass: λ_ν and M_N–scale.
  - Large neutrino mixing: Influence on the asymptotic Yukawa unification.
- $M_{string} > M_{GUT}$: Changes on the WMAP areas respect the CMMS/mSUGRA.
- INFLUENCE IN THE MODEL:
  - WMAP selects the soft-susy parameters.
  - Neutrino-data, Yukawa structure.
- Predictions for beyond the SM decays:
  - BR($L_i \rightarrow L_j\gamma$)
  - LFV decays $\chi_2 -> \chi\tau\mu$ at the LHC.

b-τ unification and m_b effects

Without considering lepton mixing, $b-\tau$ unification only possible for $\mu < 0$ and $tan\beta \sim 30-40$



SUSY correction to m_b

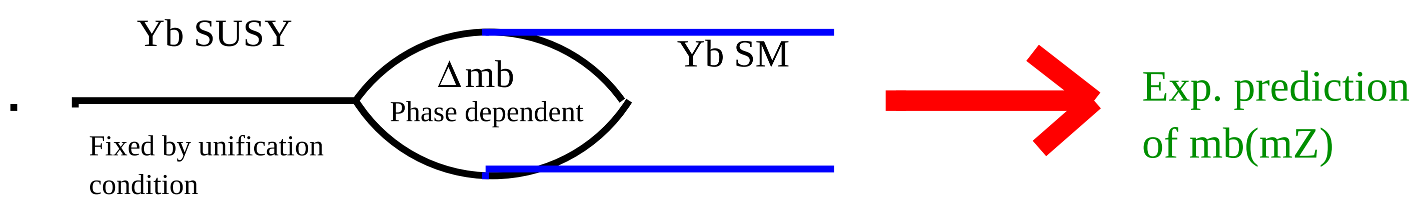
At the loop level the effective b quark coupling with the Higgs is given by:

$$-\mathcal{L}_{bbH^0} = (h_b + \delta h_b)\overline{b}_R b_L H_1^0 + \Delta h_b \overline{b}_R b_L H_2^{0*} + hc$$

with:

$$\Delta m_b = [Re(\frac{\Delta h_b}{h_b})\tan\beta + Re(\frac{\delta h_b}{h_b})]$$

It can be very large:



The $\overline{MS}$ 95% C.L. $m_b(M_Z)$ with $\alpha_s = 0.1185$ is:

$$(3.95 < m_b(m_b) < 4.55)\ \text{GeV} \longrightarrow (2.69 < m_b(M_Z) < 3.09)$$

NEUTRINO FLAVOUR OSCILLATION

- Neutrino data: By now convincing for $m_\nu \neq 0$ and physics beyond SM
- What do we know?

Atmospheric problem	Solar problem
$\Delta m^2_{atm} = \left(2.6^{+0.4}_{-0.7}\right) \times 10^{-3}$ eV^2	$\Delta m^2_{sol} = \left(8.1^{+0.5}_{-0.5}\right) \times 10^{-5}$ eV^2
$\sin^2 2\theta_{atm} > 0.90$	$\sin^2 2\theta_{sol} = \left(0.86^{+0.05}_{-0.06}\right)$

- Questions:
  1. How massive neutrinos affect Yukawa unification?
     Do they alter the predictions for the bottom mass?
  2. What are the implications for DM and LFV?

Neutrino mass: see-saw mechanism

- Let us consider right-handed neutrinos with mass $M_R \gg M_W$
- The full neutrino mass matrix becomes

$$\mathcal{M} = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^{D^T} & M_R \end{pmatrix}$$

- The physical masses are:
  1. $m_1 \equiv m_{light} \simeq \frac{\left(m_\nu^D\right)^2}{M_R}$
  2. $m_2 \simeq M_R$
- For $(m_\nu^D)_{33} \approx (200 \text{ GeV})$ $(\lambda_\nu \approx \lambda_t)$ and $M_{N_3} \approx O(10^{14} \text{ GeV})$, $m_{eff} \approx 0.05$ eV

Neutrino mass effects

RGE: $M_X \to M_R$

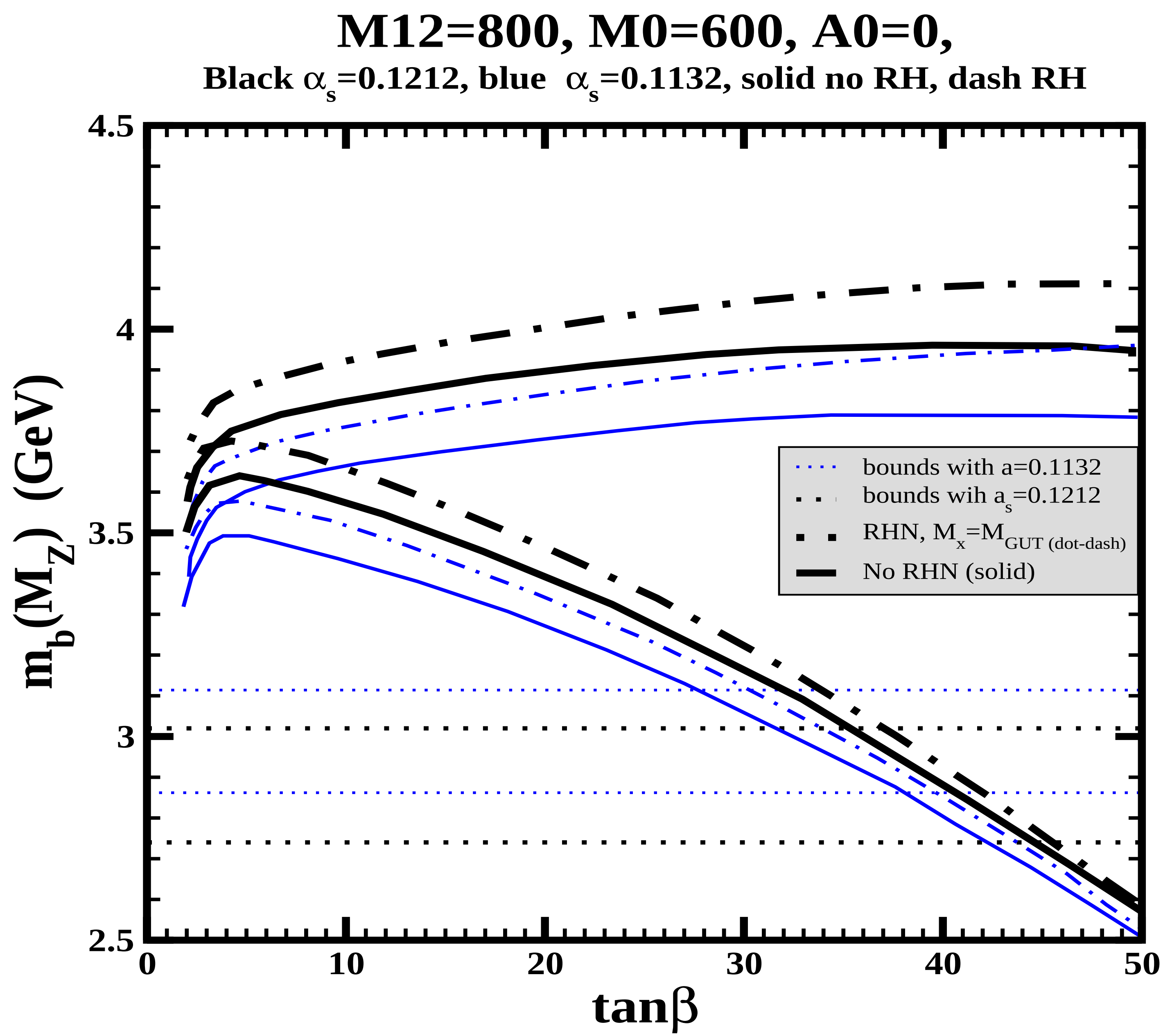
$$16\pi^2 \frac{d}{dt}\lambda_t = \left(6\lambda_t^2 + \lambda_\nu^2 - G_U\right)\lambda_t$$

$$16\pi^2 \frac{d}{dt}\lambda_\nu = \left(4\lambda_\nu^2 + 3\lambda_t^2 - G_N\right)\lambda_\nu$$

$$16\pi^2 \frac{d}{dt}\lambda_b = \left(\lambda_t^2 - G_D\right)\lambda_b$$

$$16\pi^2 \frac{d}{dt}\lambda_\tau = \left(\lambda_\nu^2 - G_E\right)\lambda_\tau$$

b-τ unification, α_s effects



SU(5) GUT's

- SUSY $SU(5)$ RH superpotential

$$\mathcal{W}_X = T^T \lambda_u T H + T^T \lambda_d \bar{F} \bar{H} + \bar{F}^T \lambda_\nu S H + S^T M_R S$$

- With the matter content: $\bar{F}$=5=(D_R^c, L), T=10=(Q,U_R^c,E_R^c).

Yukawa textures in some basis:

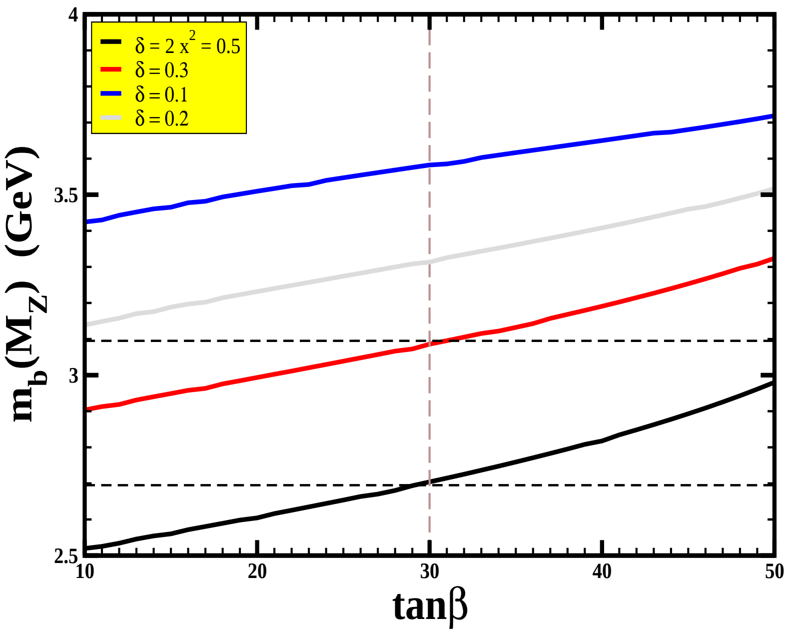
$$m_\ell^0 = m_0 \begin{pmatrix} 0 & x \\ 0 & 1 \end{pmatrix}, \quad m_{\rm D}^0 = m_0 \begin{pmatrix} 0 & 0 \\ x & 1 \end{pmatrix}$$

The unification condition

$$\frac{m_b^0}{1+x^2} = \frac{m_\tau^0}{1-x^2} \rightarrow m_b^0 = m_\tau^0 \left(1 - \underbrace{2x^2}_{\delta} + O(\delta^2) \right)$$

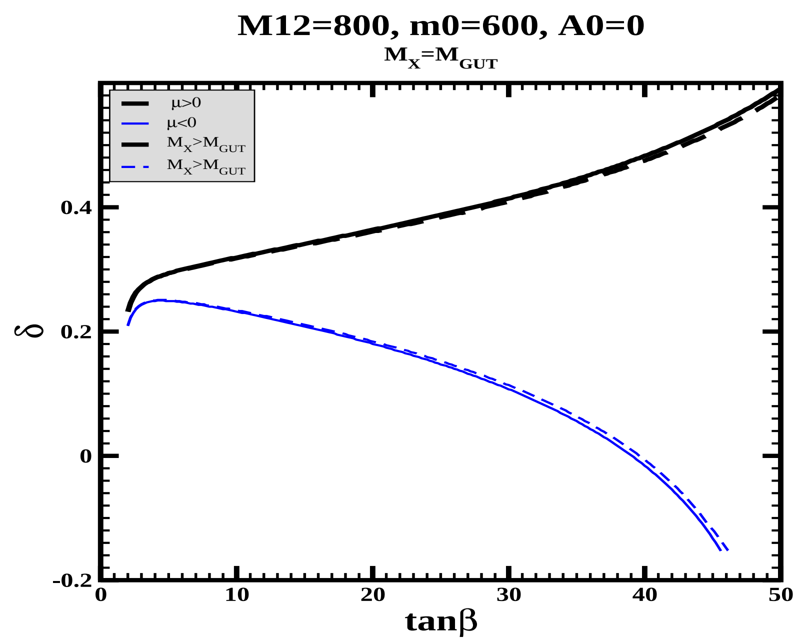
Lepton Mixing Effects

Considering lepton mixing ($\delta \neq 0$), $b - \tau$ unification possible for <u>both</u> signs of μ and a large range of $tan\beta$



Bottom mass in terms of tanβ for different values of δ.

The value of δ computed for the experimental central value of m_b in terms of tanβ.



SU(5) RGE effects

The running of the soft terms from a higher scale (M_X) to M_{GUT} introduce non universalities on the soft terms :

- $M_x \longrightarrow M_{GUT}$

$$W_{\mathrm{SU}(5)} = \frac{1}{4} f_u^{ij} \, 10_i \, 10_j \, H + \sqrt{2} f_d^{ij} \, 10_i \, \bar{5}_j \, \overline{H} + f_\nu^{ij} \, 1_i \, \bar{5}_j \, H$$

$$f_u^{ij} = f_u^{\delta},$$

$$f_d^{ij} = V_{CKM}^{*} \lambda_d^{\delta} V_{KM}^{\dagger}$$

The soft terms:

$$m_{10} \, \widetilde{10} * \widetilde{10} + m_5 \, \widetilde{5} * \widetilde{5} + \cdots$$

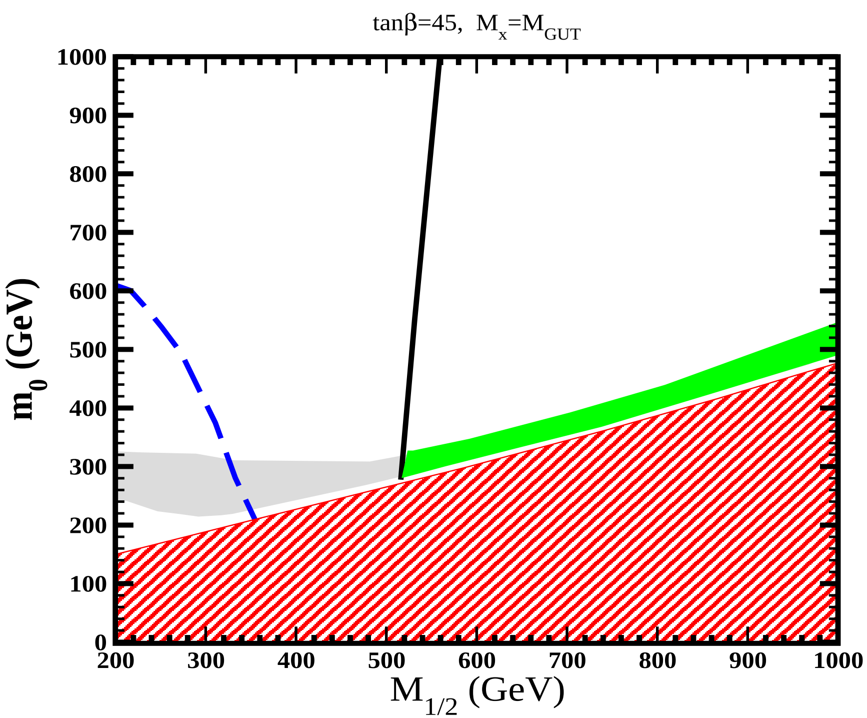
$$\widetilde{\ell}_R \text{ in } 10's \longrightarrow m^2_{\widetilde{\ell}_R} = V^{\dagger}_{\mathrm{CKM}} m^2_{10} V_{\mathrm{CKM}}$$

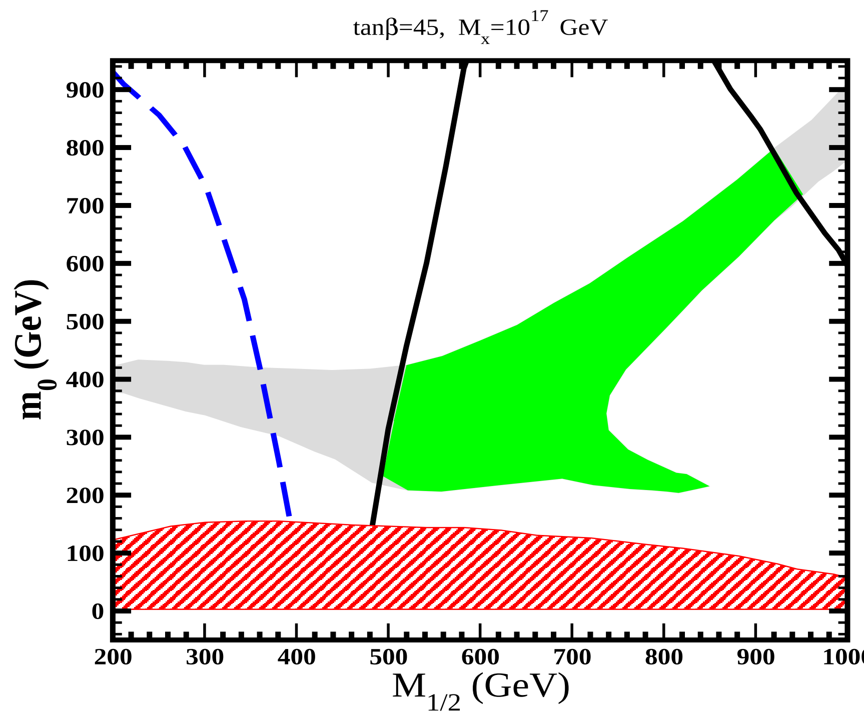
WMAP and Dark matter constraints ($\tan\beta = 45$)

We assume $\mu > 0$ and $m_b(m_Z) = 2.92$ GeV $\Rightarrow y_b = y_\tau(1-\delta),\ \delta \sim 0.42$

The favored paramter space (green) is bounded by WMAP area and $m_h < 114$ GeV

$M_X > M_{\rm GUT}$





$M_X = M_{\rm GUT}$

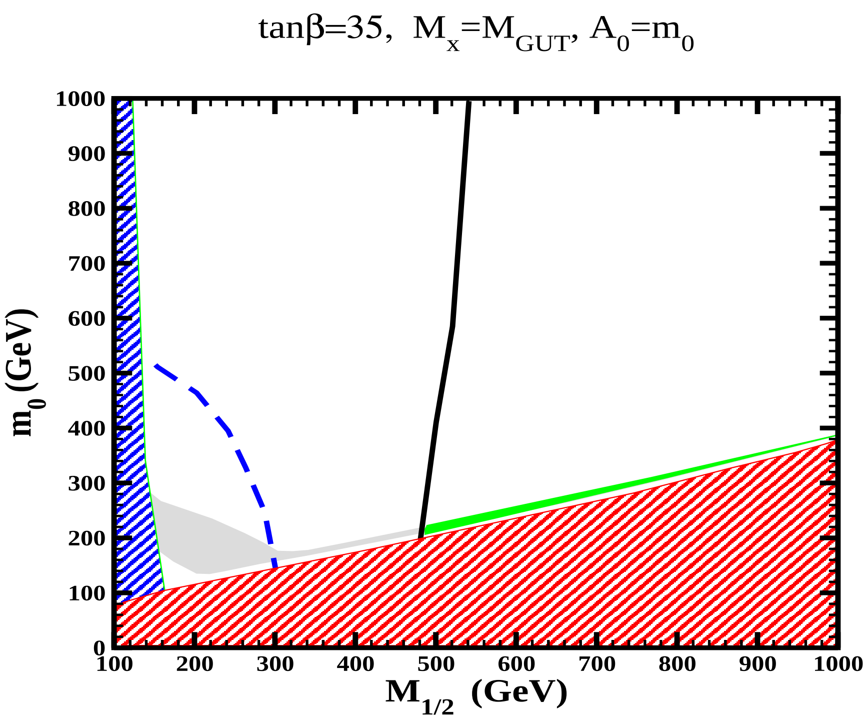
WMAP and Dark matter constraints $(\tan\beta = 35)$

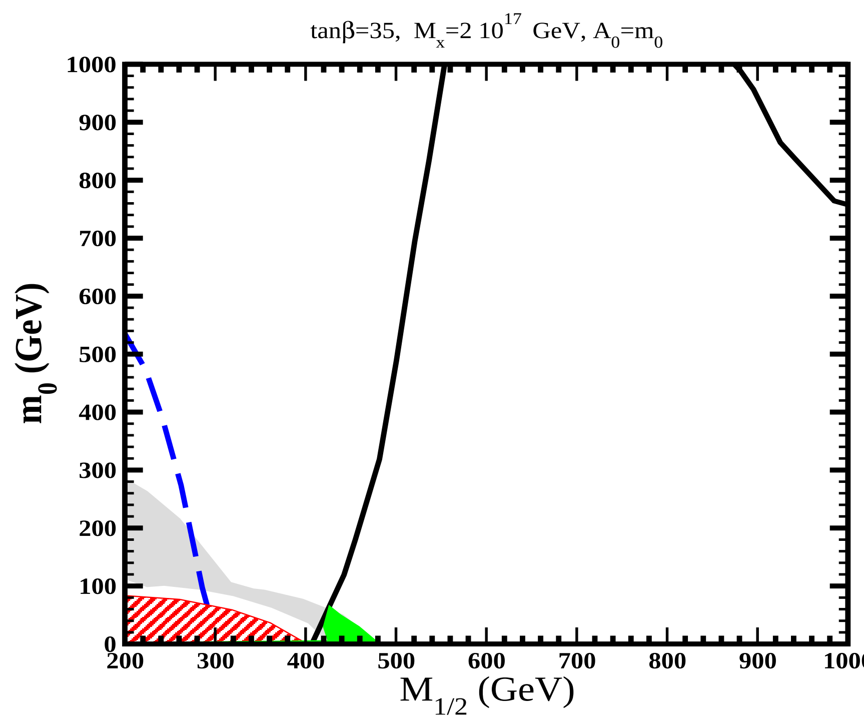
We assume $\mu > 0$ and $m_b(m_Z) = 2.92$ GeV $\Rightarrow y_b = y_\tau(1-\delta),\ \delta \sim 0.37$

The favored paramter space (green) is bounded by WMAP area and $m_h < 114$ GeV

$M_X > M_{\rm GUT}$

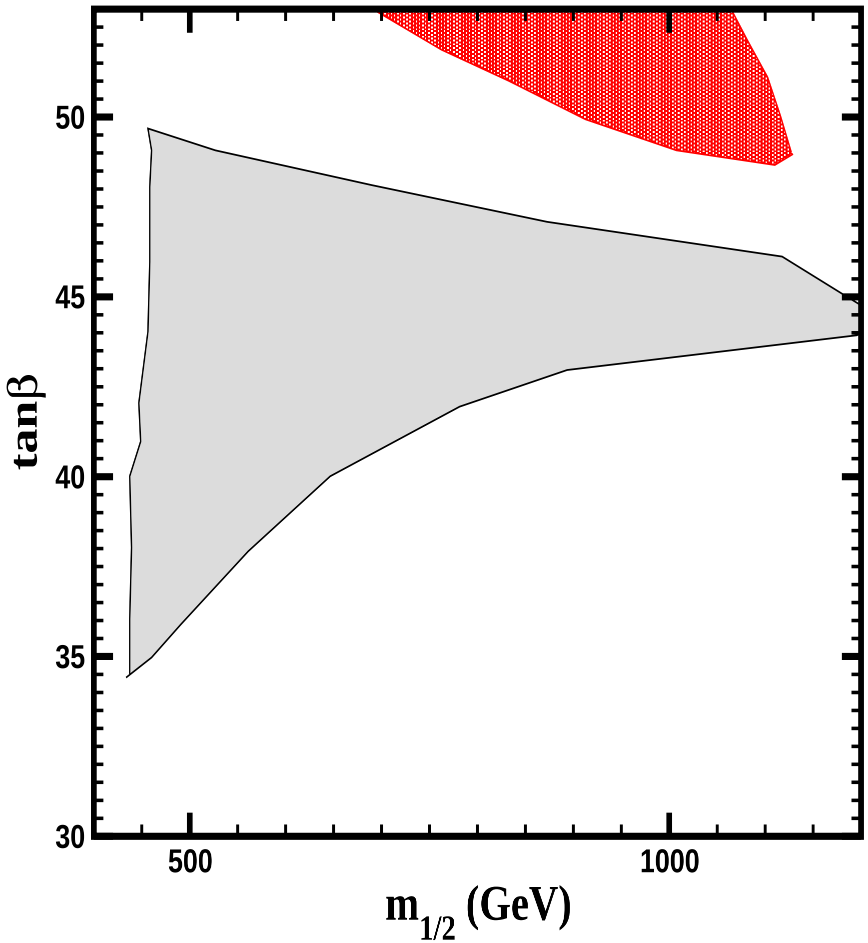
We find a lower $\tan\beta$ value (~ 33) below which the WMAP area is excluded by the m_h constraint.

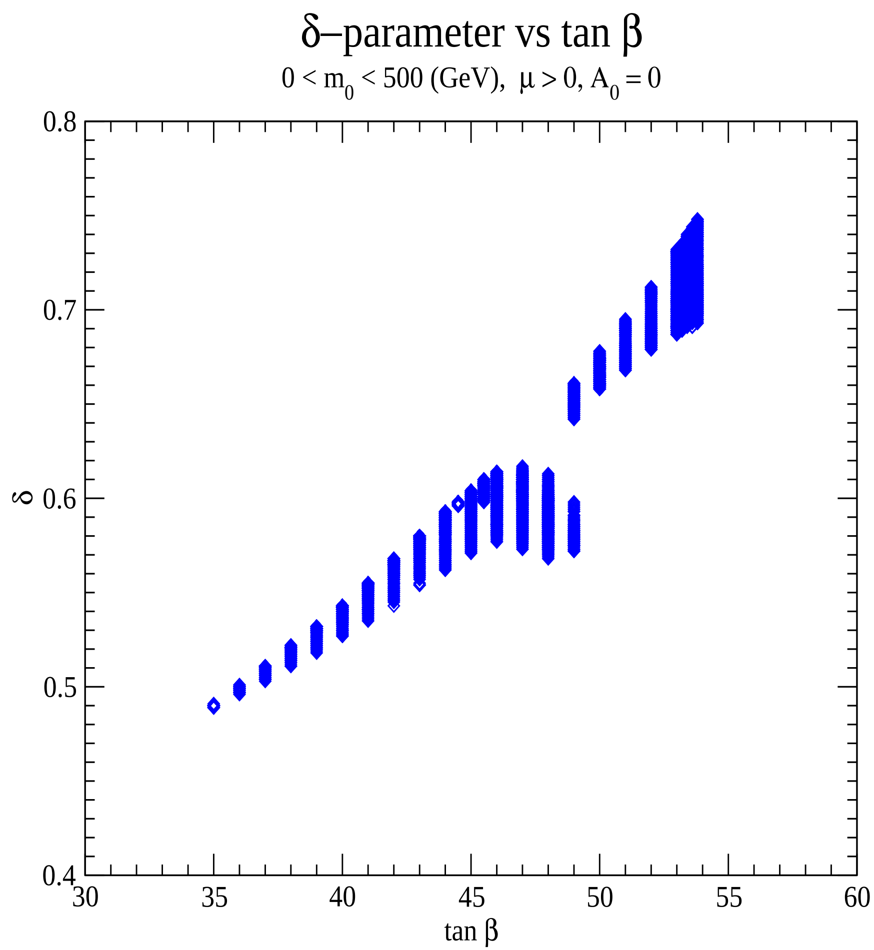




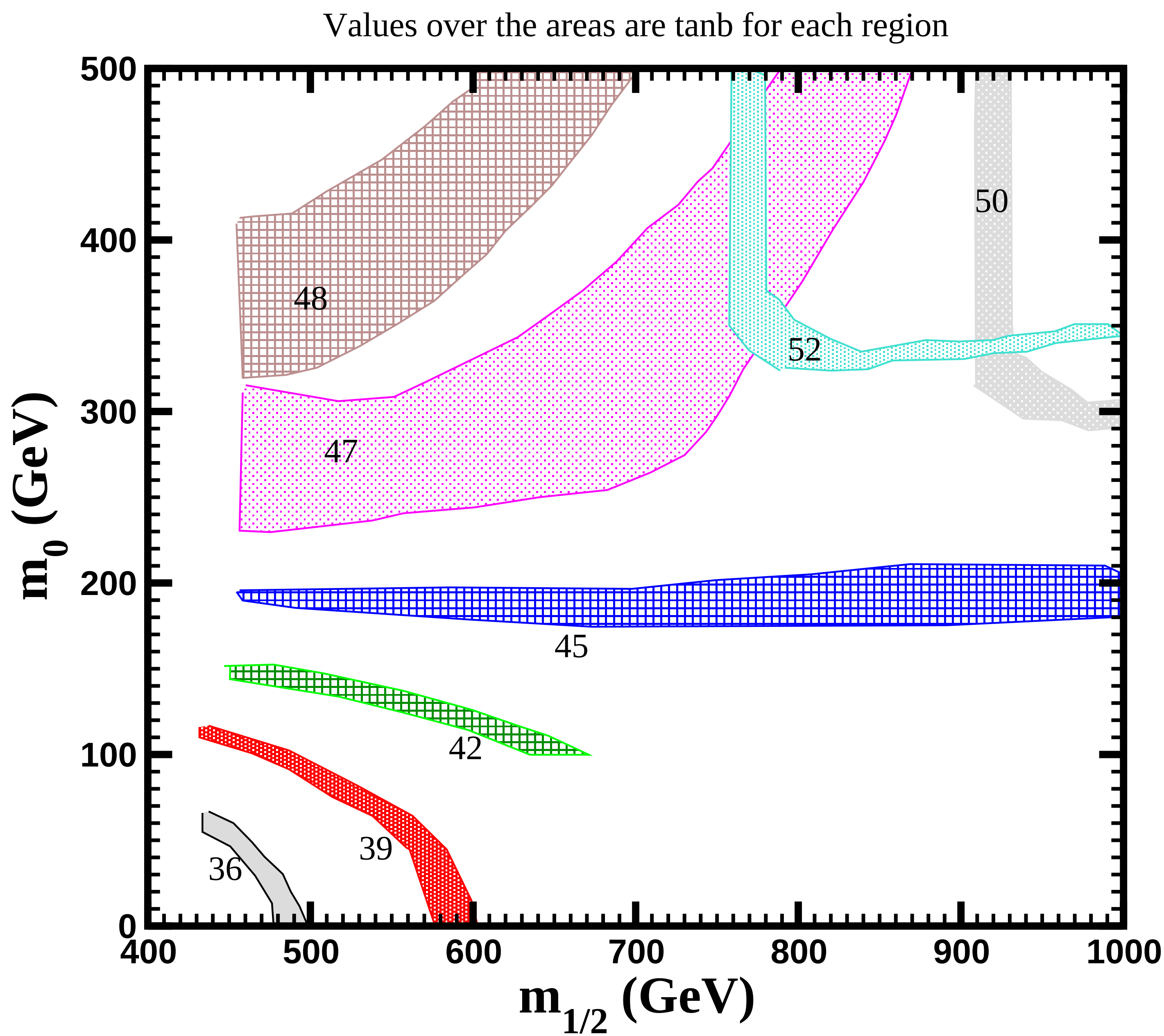
$M_X = M_{\rm GUT}$

Parameter Space $m > 0$, $A_0 > 0$





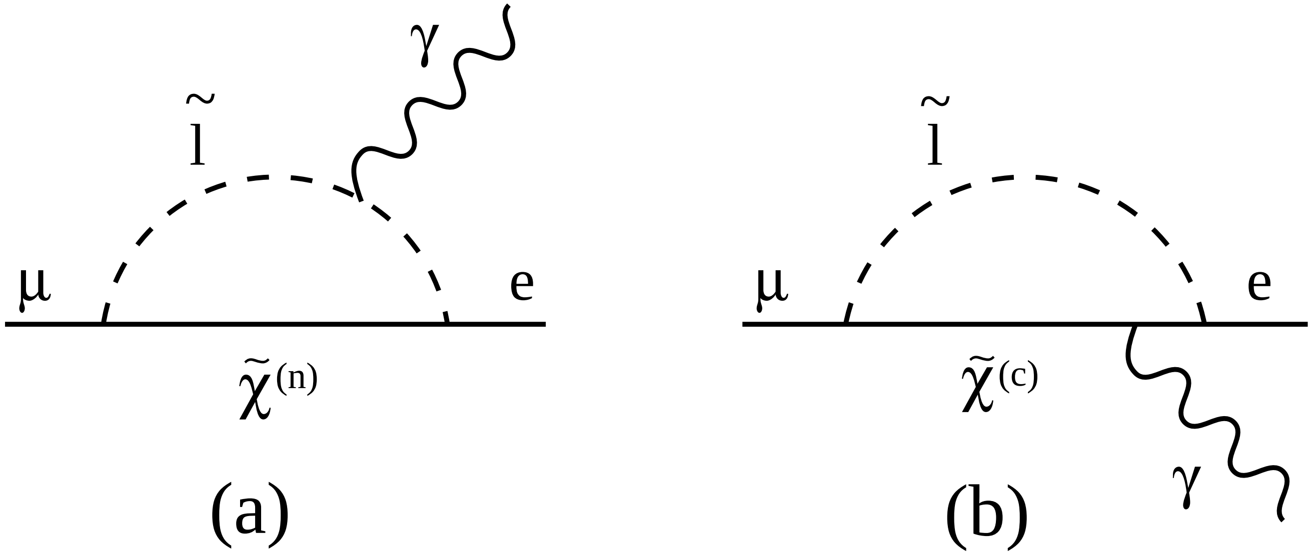
$mu > 0, A_0 = 0$



Values over the areas are tanb for each region
500
400
300
200
100
0
400
500
600
700
800
900
1000
m0 (GeV)
m1/2 (GeV)
48
47
50
52
45
42
39
36

LFV in minimal SUSY

In SUSY we can generate LFV diagrams at one loop



The magnitude of the rates depends on:

1. The mass of superparticles
2. The mixing of superparticles

For non-universality at M_{GUT}, large rates.

Massive ν & SUSY

Even if:

$$M_{\rm GUT} : m_{\tilde{\ell},\tilde{\nu}} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{RGEs} \longrightarrow \begin{pmatrix} 1 & \star & \star \\ \star & 1 & \star \\ \star & \star & 1 \end{pmatrix}$$

- RGEs for the charged-lepton mass matrix

$$t\,\frac{d}{dt}\,(m^2_{\tilde{\ell}})^j_i = \frac{1}{16\pi^2}\left\{\left(m^2_{\tilde{\ell}}\lambda^\dagger_\ell\lambda_\ell\right)^j_i + \left(m^2_{\tilde{\nu}}\lambda^\dagger_\nu\lambda_\nu\right)^j_i + \cdots\right\}$$

The corrections in the basis where $(\lambda^\dagger_\ell\lambda_\ell)^j_i$ is diagonal, are:

$$\delta m^2_{\tilde{\ell}} \propto \frac{1}{16\pi}\ln\frac{M_{\rm GUT}}{M_N}\lambda^\dagger_\nu\lambda_\nu m^2_{SUSY}$$

- $M_{GUT} \to M_R$

$$W_{\mathrm{MSSM}+\nu_R} = Q^T\ f_u^\delta\, U\, H_2 + Q^T \left(V_{CKM}^\dagger f_d^\delta\right) D\, H_1 + L^T \left(V_{KM}^* f_\ell^\delta\right) E\, H_1 + L^T f_\nu^\delta\, N\, H_2$$

Remember that the $V_{KM} = V_\nu^+ \cdot V_l$ where $V_\nu^+ \cdot f_\nu^+ f_\nu \cdot V_\nu = (f_\nu^\delta)^2$ and $V_l^+ \cdot f_l^+ f_l \cdot V_l = (f_l^\delta)^2$. (Does not involve the RH neutrinos like the V_{NMS}). At scale M_R, the diagonal charged lepton Yukawa implies:

$$L^* \left(m_l^2\right)^{diag} L \to L^* \left[V_{KM}^\dagger \cdot \left(m_l^2\right)^{diag} \cdot V_{KM}\right] L$$

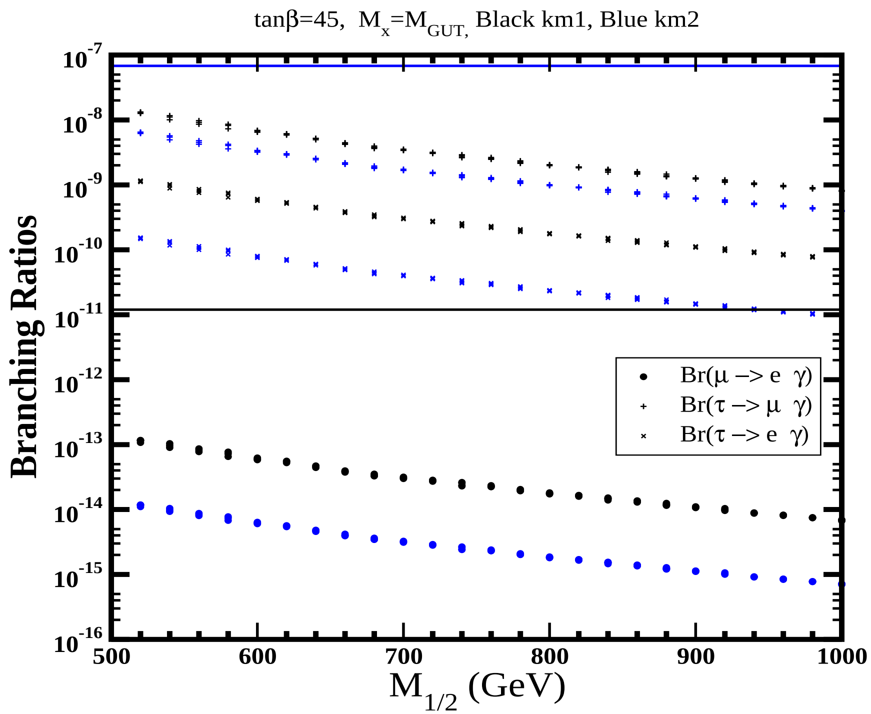
Suitable textures to explain ν data are predicted in $SU(5)$ models with an addtional U(1)-family symmetry. By taking 2 examples from *JHEP0707 (2007) 052* we find:

$$V_{KM}^{(1)} = \begin{pmatrix} 0.94 & 0.35 & -0.0114 \\ -0.35 & 0.92 & -0.165 \\ -0.047 & 0.16 & 0.97 \end{pmatrix}$$

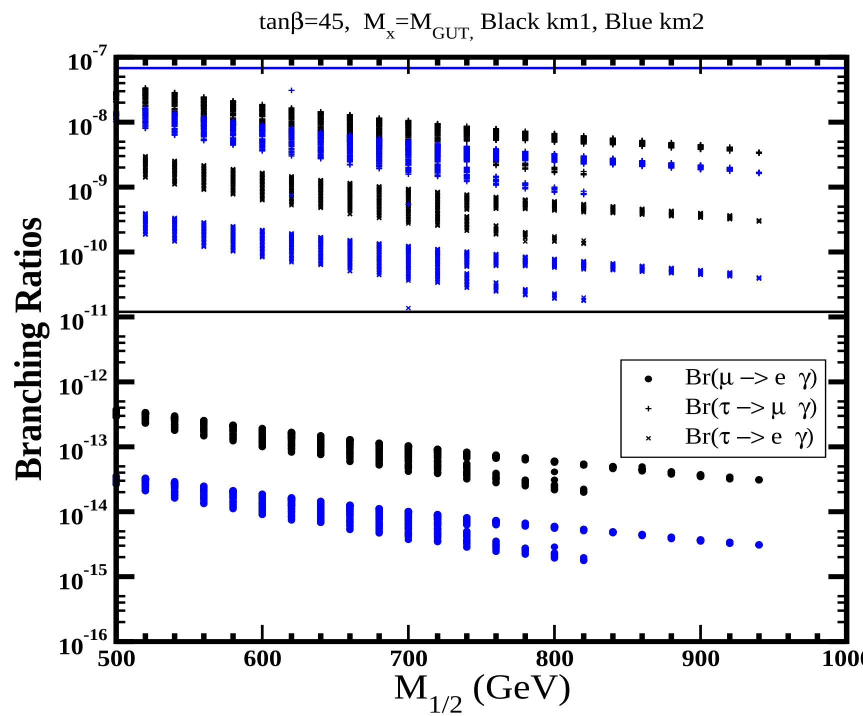
$$V_{KM}^{(2)} = \begin{pmatrix} 0.97 & 0.16 & -0.034 \\ -0.16 & 0.98 & 0.105 \\ -0.017 & 0.11 & 0.99 \end{pmatrix}$$

Branching ratios for the allowed parameter space ($\tan\beta = 45$)

We consider values of m_0 on the area satisfying WMAP and $m_h < 114$ GeV.

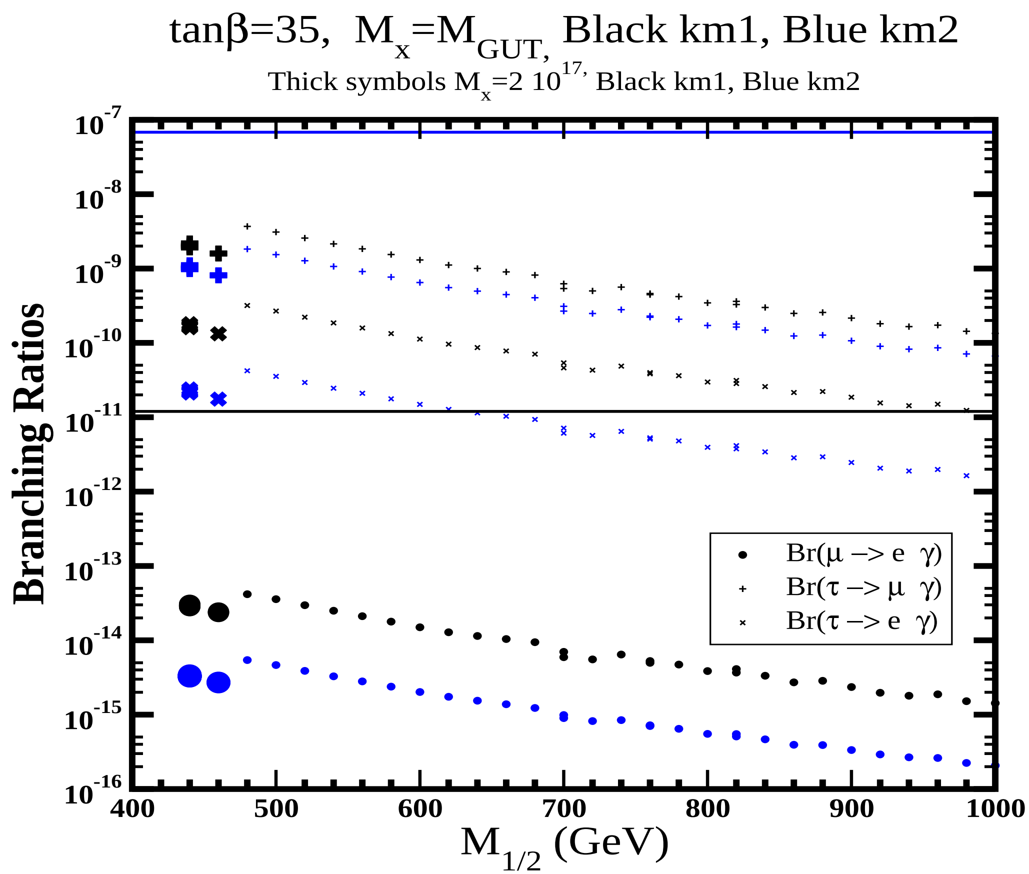


$M_X > M_{\rm GUT}$



$M_X = M_{\rm GUT}$

Branching ratios for the allowed parameter space ($\tan\beta = 35$)



$M_X = M_{\rm GUT}$ (small symbols) $M_X = 2 \cdot 10^{17}$ (big symbols)

$\chi_2 \to \chi + \tau^\pm + \tau^\mp$ at LHC.

- On-shell slepton production:

$$BR(\chi_2 \to \chi\tau^\pm\mu^\mp) = \sum_{i=1}^{3} BR(\chi_2 \to \tilde{l}_i\mu) BR(\tilde{l}_i \to \tau\chi) + BR(\chi_2 \to \tilde{l}_i\tau) BR(\tilde{l}_i \to \mu\chi)$$

- the signal in the τ channel to be optimal is definded by the following:

  (i) $m_{\chi_2^0} > m_{\tilde{\tau}} > m_\chi^0$ (on-shell condition)

  (ii) $m_{\tilde{\tau}} >> m_\chi^0$ (hadronised τs in the final state)

  (iii) Moderate values of m_χ^0 (phase space and luminosity considerations).

Conclusions

- The SU(5) unfication condition has been revisited, including a *see–saw* mechanism in the MSSM. By only asssuming hierarchical Yukawa couplings, the problem of the m_b prediction for $\mu > 0$ is not solved.
- The assumption of a sizeable 2-3 flavour mixing in the lepton sector
  1. allows unification for $\mu > 0$
  2. enhances the allowed range of $\tan\beta$ where $y_b - y_\tau$ unification is possible.
- Assuming a $SU(5)$ RGE running from $M_x > M_{GUT}$ the WMAP favoured parameter picture changes, respect the typical mSUGRA scenario.
- For the cosmologically preferred region, LFV close to the current experimental bounds is also predicted.
- The possible detection of LFV in SUSY decays for space of parameters favored by WMAP is under investigation.