Integrability in Gauge and String Theory: 2010

Summary Talk

recent years: new evidence of key role of integrability-based methods in solving important problems of theoretical physics

General aims:

• understand quantum gauge theories at any coupling: dimensions, correlators, scattering amplitudes, Wilson loops, hidden symmetries, ...

• understand quantum string theories in RR backgrounds, prove AdS/CFT conjecture(s): AdS_5/CFT_4, ...

• develop new methods based on integrability that may have broader applications

Talks on gauge theory:

Lipatov, Kristjansen, Rastelli, Maldacena, Wyllard, Ferro, Johansson, Henn, Spradlin, Sokatchev

Talks on integrability methods:

general: Kulish, Smirnov, Staudacher, Beisert

applied to AdS/CFT: Kazakov, Tateo, Arutyunov, Lukowski, Hegedus

Talks on string theory:

Bykov, Vieira, Maldacena, Janik, Roiban

Planar AdS/CFT duality: Spectrum

I. Spectrum of "long" operators = "semiclassical" string states

Asymptotic Bethe Ansatz vs perturbative string theory

• now checked beyond doubt via highly non-trivial comparison, earlier contradictions resolved [Roiban]

• first-principle derivation of ABA from string theory?

R-matrix approach to classical superstring σ-mode, QISM ?

(non-ultralocality related to twist by Zhukovsky map,

non skew-symmetric R-matrix, di-algebra)

[Vicedo]

• $AdS_4 \times CP^3$ case: IIA superstring sigma-model in classical spinning string background

effective theory for massless excitations:

an (integrable?) CP^3 sigma-model with fermions [Bykov]

II. Spectrum of “short” operators = all quantum string states

Thermodynamic Bethe Ansatz : tool to derive the spectrum
important progress in understanding non-trivial structure of TBA

But still very complicated analytic structure
(in particular, due to lack of relativistic invariance)
progress in sl(2) case so far

“The truth is rarely pure and never simple”. Oscar Wilde
should we accept this ?

TBA is based on several assumptions
(e.g. not really justified at weak coupling:
1-st principles understanding of wrapping?)
direct tests at weak and strong coupling are necessary

• Y-system, Hirota integrable dynamics, Wronskian solution, semiclassical solution in AdS/CFT context [Kazakov]

• study of analytic properties of the Y- functions
derivation of TBA equations: Y-system + discontinuity relations
$\rightarrow$ set of local functional constraints in integral form [Tateo]

• subtleties of excited states and their analytic properties from TBA for the $AdS_5 \times S^5$ mirror model; use of sl(2) TBA equations to rederive 5-loop Konishi dimension [Arutyunov]

• 5-loop twist-2 dimension for any spin: test of TBA
use reciprocity to get ABA part; Luscher to get wrapping
checks against BFKL and double-log constraints [Lukowski]

• analytic test of TBA in weak coupling limit:
5-loop anomalous dimensions of twist-2 operators
agree with obtained previously from Luscher formulae [Hegedus]

Strong-coupling test of TBA against string theory for Konishi state?

"It is a fine thing to be honest, but it is also very important to be right"
Winston Churchill

Still open question about subleading terms
in strong-coupling expansion of Konishi dimension:

$$\gamma(\lambda \gg 1) = 2\sqrt[4]{\lambda} + b_0 + \frac{b_1}{\sqrt[4]{\lambda}} + \frac{b_2}{(\sqrt[4]{\lambda})^2} + \frac{b_3}{(\sqrt[4]{\lambda})^3} + ...$$

TBA: $b_1 \approx 2$ [Gromov, Kazakov, Vieira, 2009; Frolov, 2010]
Semiclassical string theory argument: $b_1 = 1$ [Roiban, AT 2009]
based on several assumptions (order of limits, etc.)
Need to push further perturbative string theory computations
(near flat space expansion, AdS l.c. gauge, ...)
as well as develop analytic methods on TBA side

Semiclassical string theory:
universality of b_1? integer for rational solutions
but not for elliptic ones?
Folded spinning string and pulsating string cases
[Tirziu, AT 2008; Beccaria, Dunne, Forini, Pawellek, AT 2010;
Beccaria, Dunne, Macorini, Tirziu, AT, in progress]

Folded spinning string in AdS_3

$$E = \sqrt{2S\sqrt{\lambda}}\left(1 + \frac{\frac{3}{8}S + \frac{3}{2} - 4\log 2}{\sqrt{\lambda}} + \dots\right) + 1 + \dots$$

Folded spinning string in $\mathbb{R} \times S^2$

$$E = \sqrt{2J\sqrt{\lambda}}\left(1 + \frac{\frac{1}{8}J + 2 - 4\log 2}{\sqrt{\lambda}} + \dots\right) + 2 + \dots$$

Pulsating string in AdS_3

$$E = \sqrt{2N\sqrt{\lambda}}\left(1 + \frac{\frac{5}{8}N + \frac{5}{2} - 4\log 2}{\sqrt{\lambda}} + \dots\right) + 1 + \dots$$

Pulsating string in $\mathbb{R} \times S^2$

$$E = \sqrt{2N\sqrt{\lambda}}\left(1 + \frac{-\frac{1}{8}N + 1 - 4\log 2}{\sqrt{\lambda}} + \dots\right) + 2 + \dots$$

Relation to Konishi states: $J = 2,\ S = 2$, ... ?

“Null” Wilson loops / gluon amplitudes

Last year: remarkable progress in Alday-Maldacena program use of integrability of string theory to determine (via relation to null Wilson loops) leading strong coupling contributions to gluon scattering amplitudes

“However beautiful the strategy, you should occasionally look at the results”. Winston Churchill

- area of surfaces that end on a null polygon at AdS boundary as function of conformal cross ratios characterizing polygon: free energy determined by a TBA eq. for a relativistic system (any number of gluons, in any kinematic configuration) [Vieira]
- special collinear limit: OPE-type expansion

subleading corrections governed by excitations of high spin op’s (excitations of flux tube between two Wilson lines)

universality: any gauge CFT, any coupling, in any dimension

N=4 SYM: checks at strong coupling and 2 loops at weak coupling (hexagon); predictions at higher loops [Maldacena]

Gauge theory scattering amplitudes

integrability, hidden symmetries, new methods, ...

• Integrability of high energy amplitudes in N=4 SYM:
multi-particle planar scattering amplitudes in the multi-Regge kinematics– high energy behavior due to gluon composite states
Hamiltonian for these states in leading log approximation
= local Hamiltonian of an integrable open spin chain [Lipatov]

• Yangian invariance of scattering amplitudes in N=4 SYM
written in terms of Grassmannian integral: proved directly using explicit form of Yangian level-one generators.
Yangian symmetry fixes uniquely cyclic structure
of the form integrated over the Grassmannian [Ferro]

• Scattering amplitudes on the Coulomb branch of N=4 SYM:
regularization of IR ∞'s using scalar expectation values;
conceptual and practical advantages over dim. reg.:
dual conf. symm. exact; restriction of basis of integrals.
5-loop diagram test of cusp anomaly ? [Henn]

• duality between color and kinematics in gauge theory amplitudes → nontrivial relations between color-ordered partial tree amplitudes → construction of gravity amplitudes as double copy of gauge theory kinematic factors clarifying KLT relations; evidence that color-kinematics duality is valid at the quantum level [Johansson]

• Motiv-ation: analytic formula for 2-loop 6-point MHV remainder function in N=4 SYM — simplest non-trivial case (= two-loop light-like hexagon Wilson loop)
classical polylog of cross-ratios of momentum twistor invariants [Spradlin]

• superconformal and dual superconformal symmetries of N=4 SYM scattering amplitudes have to be complemented with analytic properties to fully fix the tree S-matrix;
how symm's broken at loop level; implications?
amplitudes in twistor space (Grassmannian invariants) [Sokatchev]

Integrability: general methods

"Although personally I am quite content with existing explosives,
I feel we must not stand in the path of improvement". Winston Churchill

• Quantum integrable spin systems related to quantum groups:
generalized Schur - Weyl duality [Kulish]
• Hidden fermionic structure of integrable models:
use it to compute one-point functions of the primary fields
and their descendants for Euclidean SG model on cylinder:
find complete asymptotic series for 2-point functions [Smirnov]

• Why Q ?

Q-operator: powerful tool to diagonalize integrable models

novel construction of Baxter's Q-operator for $sl(n|m)$ systems

Possible importance of Q for physical interpretation of

Y-system of AdS/CFT [Staudacher]

• Why q?

Quantum deformations of magnon/w-sheet S-matrix

potentially important to reveal underlying algebraic structure

deformation of Yangian for superalgebra $sl(2|2)$

novel classical r-matrix of trigonometric kind

should find interesting applications (e.g. in SG-type models

based on massive deformation of gWZW: Pohlmeyer reduction)

[Beisert]

Generalizations/Extensions

• relaxing planar limit: non-planarity and integrability?
non-planar ABJ(M), integrability and parity
N = 4 SYM with gauge group SO(N):
leading 1/N-corrections are described by single spin chain
search for integrability by standard BA methods ?
tests against dual string theory on the orientifold $AdS_5 \times RP^5$?
[Kristjansen]

• relaxing $N = 4$ susy: N=2 superconformal theory ($N_f = 2N_c$)
integrability in Veneziano limit?
limiting case of orbifold of N=4 SYM
one-loop dilation operator in the scalar sector of
N=2 superconformal quiver with $G = SU(N_c) \times SU(N_{\check{c}})$ [Rastelli]

• relations between 4d N=2 quiver gauge theories, 2d conformal
Toda field theories, topological string theories
and quiver matrix models. Surface operators in gauge theories
and their dual counterparts [Wyllard]

• correlation functions of operators dual to classical large spin string states [Janik]

strong coupling limit of gauge theory correlators
or vertex op. correlators?
Problem: find world-surfaces attached to
euclidean boundary at specific points (vertex ope
has solution in AdS_3 case described by $SL(2)$ WZW model
(cf. uniformization problem in Liouville theory)
WZW eqs. are explicitly solvable
same is in AdS_5 case due to integrability
remains to be seen ...

Conclusions

Healthy and steady progress, stimulating conference

breadth of ideas, methods, models, dualities, relations, ...

Directions and Open Problems

• algebraic ideas should bring some fruit:

bridge the gap between SYM and SM (beyond Y/Yangian ? ....)

• reformulation of TBA? Analytic solution at strong coupling

• Pohlmeyer reduction – TBA for a Lorentz-invariant system ?

solution of generalized SG-type models

(mass. deformation of gWZW, Hollowood, Miramontes '10)

• integrability of string sigma-model: further implications

for WL's / amplitudes, correlation functions, ...

• generalizations: $AdS_n \times M^k$ models, (β-) deformations, ...

• less susy, non-critical strings, non-planar, ...

Predictions

- we will see a lot of progress in these directions during next year
- Brazil–Netherlands: ?:?

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Future Conference

2011: Perimeter Institute (3rd week of August)