An Operator Product Expansion for null Polygonal Wilson loops

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Integrability in Gauge and String Theory
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Collaborators

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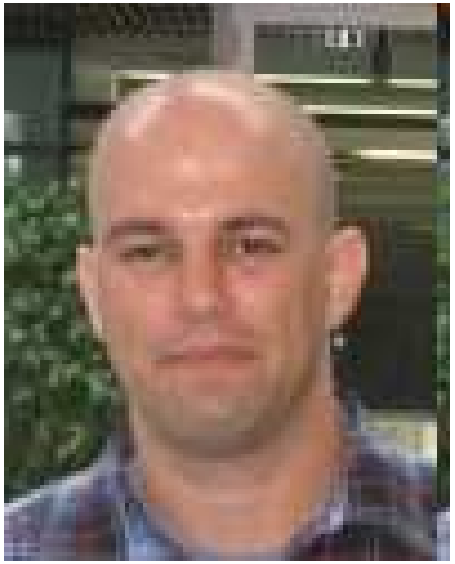

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Pedro Vieira

Motivation

- From integrability → Spectrum
- Other observables?. Correlation functions, Wilson loops, amplitudes…
- We consider Wilson loops / Amplitudes
- We know the weak and strong coupling answers. How do we go between them?

- We will show how to compute the answer in one corner of parameter space for all coupling.

- It is a simple analog of the ordinary operator product expansion, but for Wilson loops.

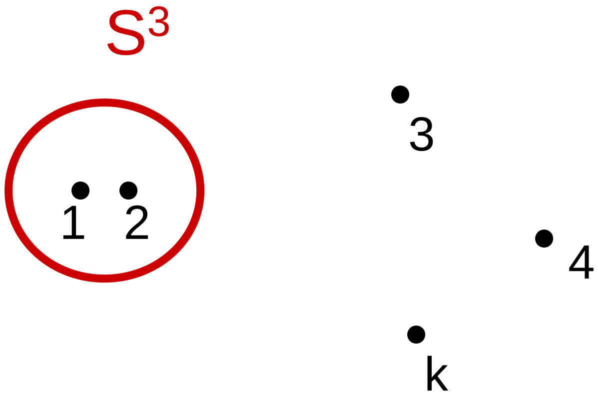
- Wilson loops with null edges are eminently lorentzian observables. Understanding this expansion could be useful for other lorentzian observables.

- The OPE expansion we derive is valid in any CFT which has a conserved electric flux.

Plan

- The ordinary OPE
- Symmetries of null lines
- Families of Wilson loops
- States that propagate
- The form of the OPE
- Checks at weak and strong coupling
- Predictions for all coupling

The ordinary OPE



$$\langle O_1 O_2 \cdots O_k \rangle$$

- We surround operators 1 and 2 with a 3-sphere.
- We have some state propagating → expand it in terms of energy eigenstates
- States on the sphere are in correspondence with local operators.

- Symmetries : We imagine acting with a dilatation on 1 & 2 (but not the rest)
  We get a family of points depending on t

$$\langle O_1 O_2 \cdots O_k \rangle \sim \sum_n e^{-tE_n} C_{12n} C_{n3\cdots k}$$

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- Euclidean time evolution

- Discrete sum → discrete spectrum of dimensions of operators

- Known dimensions → constraints on the functions that appear.

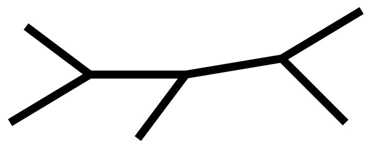
- We could surround more points by the 3-sphere and have similar expansions

- We can do it in many possible “channels”

- Consistency of the expansion in all channels → should determine the function

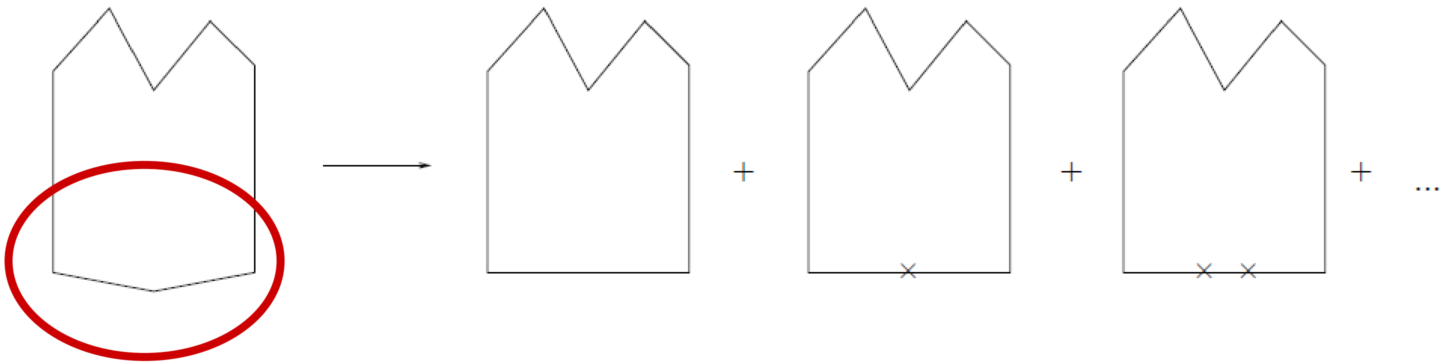
Bootstrap: Polyakov
Belavin, Polyakov, Zamolodchikov

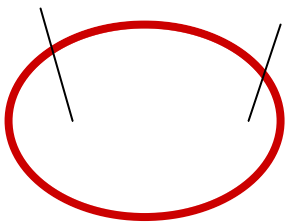
- Convergent expansion (finite radius of convergence)



Basic object is the three point function.

Wilson loops



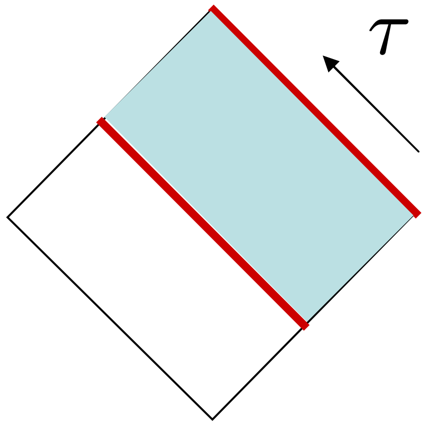


States : defined on a sphere with null lines

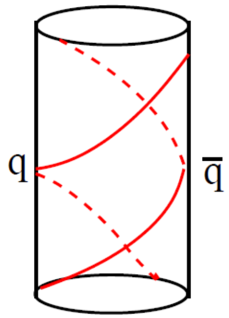
Colinear limit, including subleading terms.

Two null lines

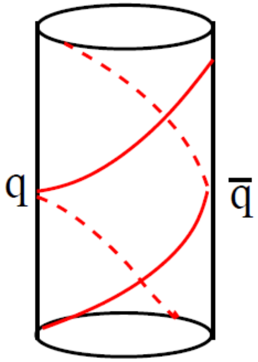
-Two generic null lines → all the same up to symmetries
- Symmetries preserved by the null lines: SL(2,R) x R x SO(2)



- Map the two lines to an $R^{1,1}$ subspace
- Lines lie along x^-
- SL(2,R) acts on x^-
- R is essentially dilations on x^+
- SO(2) rotates the transverse 2d space



- 3-sphere and two null lines.
- Null lines → null Wilson lines in the fundamental and anti- fundamental
- Color electric flux between the lines
- Flux breaks the SL(2,R) symmetry into R .



This picture also appeared in high spin operators.

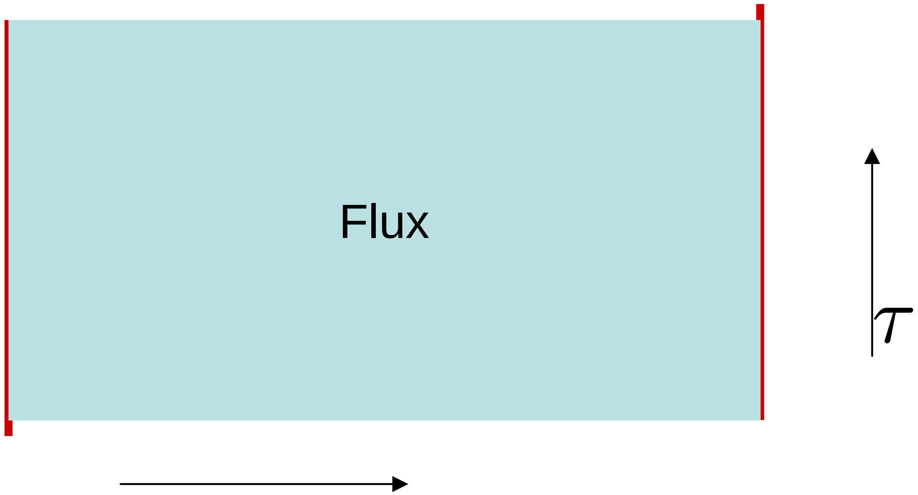
$$Tr[\Phi \partial^S \Phi]$$

As S → Infinity , we get two null Wilson lines

$\partial_\tau = \Delta - S$ = twist

In suitable coordinates we get:

Alday & JM



σ , $\Delta\sigma \sim \log S$

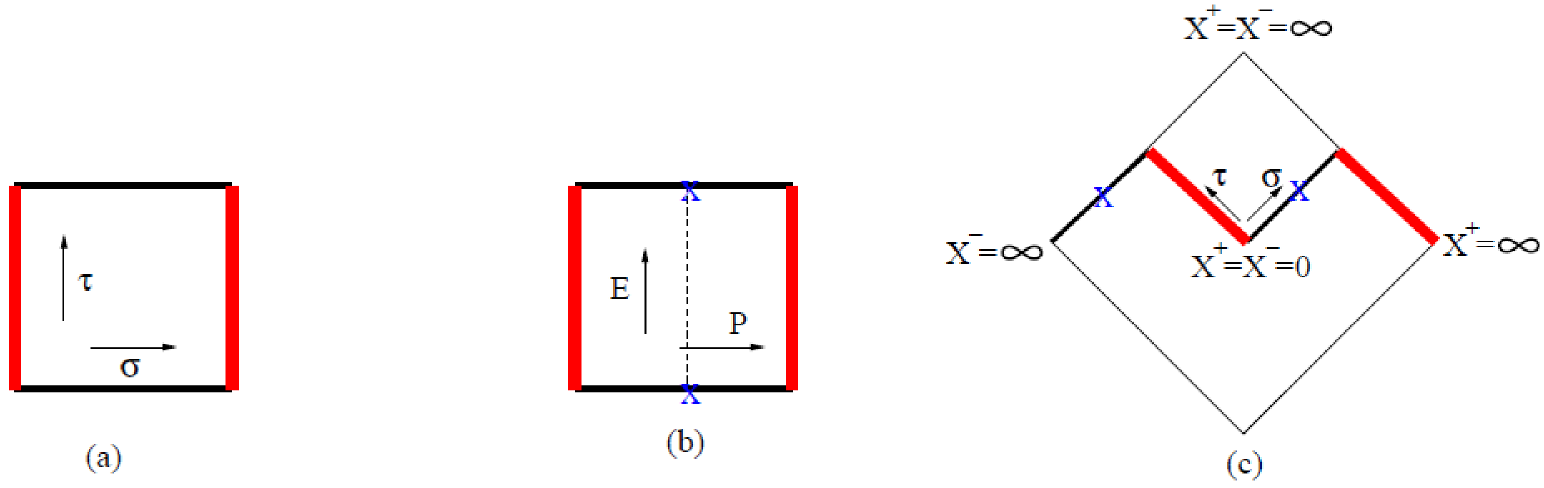
$\Gamma_{cusp}(\lambda)$ is the energy density of this flux.

τ is the time coordinate, conjugate to twist

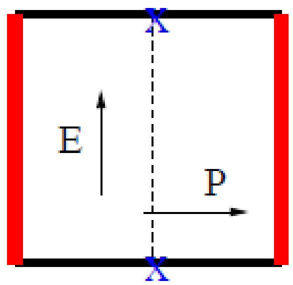
σ is the space coordinate conjugate to the extra noncompact symmetry

- So far we described the ground state.
- The ground state is the only state that propagates in the square wilson loop
- We should also consider excited states.
- In the planar theory → planar excitations only.
- Excitations of the flux tube
- Particles propagating, whose properties are modified by the presence of flux
- Viewed as insertions of operators along a null line
- Extra insertions of fields on the high spin operator.
- New vacuum: Sea of derivatives. We get impurities along the sea of derivatives.

$$Tr[\Phi\partial^n F \partial^{S-n}\Phi] \ , \qquad \text{or} \qquad \cdots\partial\partial\partial\partial\partial F \partial\partial\partial\partial\cdots$$



Properties of the excitations



- At weak coupling excitations are characterized by their "twist" = 1,2,3

-They get a correction

$$\Delta - S = E = 1 + \lambda\gamma(p) + o(\lambda^2)$$

- The twist one insertions are six scalars and two F's plus eight fermions.

-At strong coupling there are different regions depending on the momentum. The analog of the BMN region gives a relativistic dispersion relation. There is also a "giant hole" region.

Frolov Tseytlin Dorey Losi

- B. Basso computed the exact dispersion relation for some of the simplest impurities. (To appear)

Goldstone particles

Fermions with p=0 correspond to the goldstone fermions of the supersymmetries broken by the flux.

$$\epsilon(p=0)=1$$

There are bosonic modes with

$$\epsilon(p=\pm i)=1$$

This, together with relativistic invariance, gives fixes the strong coupling worldsheet spectrum.

Summary
States propagating

- Ground state. Just flux along an infinite non-compact direction.

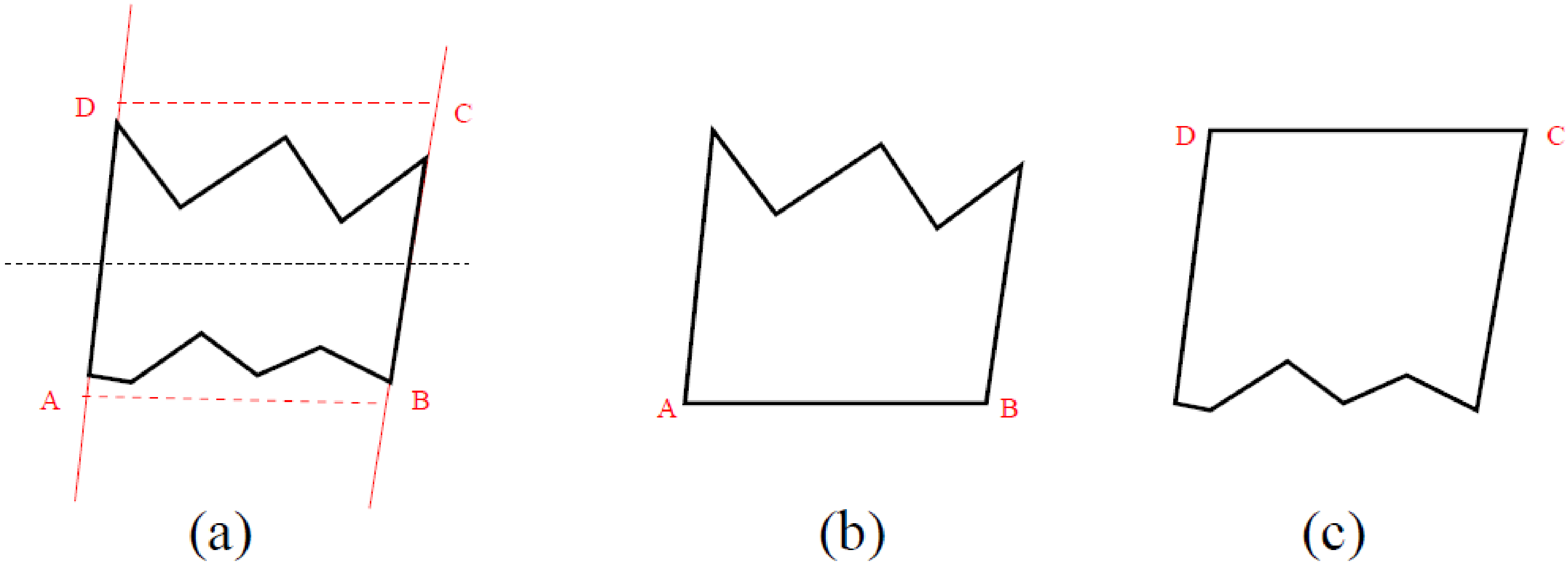
- Energy density is the cusp anomalous dimension.

- Excitations: particles propagating along this flux.

- Dispersion relation. $\epsilon(p, \lambda)$

- All states → just multiparticle states. We could have bound states, etc..

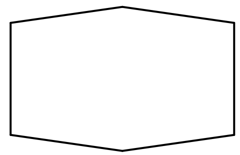
A family of Polygons



- Choose two segments of the polygon.
- Define a reference square ABCD
- Act with symmetries on the bottom side of the polygon.
- Symmetries involve three parameters: $\tau, \ \sigma, \ \phi$

Example: Hexagon

Three cross ratios.



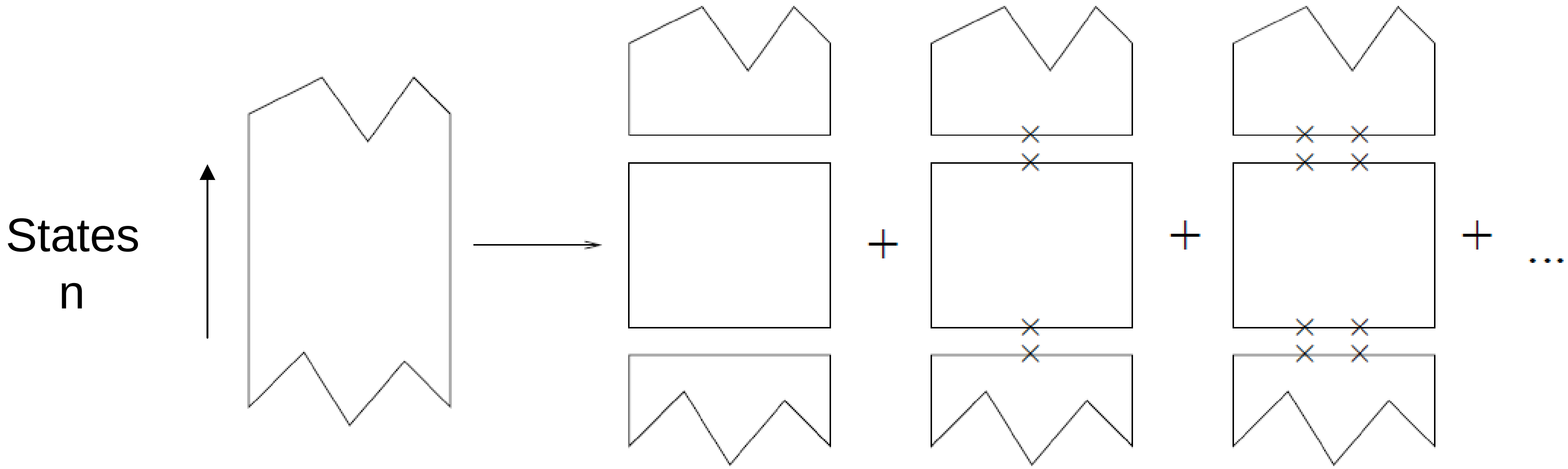
Three explicit symmetries in OPE.

$$u_2 = \frac{1}{\cosh^2 \tau}$$

$$u_1 = \frac{e^{\sigma} \sinh \tau \tanh \tau}{2(-\cos\phi + \cosh\tau \cosh\sigma)}$$

$$u_3 = \frac{e^{-\sigma} \sinh \tau \tanh \tau}{2(-\cos\phi + \cosh\tau \cosh\sigma)}$$

$$\langle W \rangle = \int dn e^{-\tau E_n + i p_n \sigma + i m_n \phi} C_n$$



Divergencies

- There are UV divergencies in the Wilson loop. These break the symmetries.

- Violation is understood

- Anomalous Ward identities.

Drummond, Korchemsky
Sokatchev

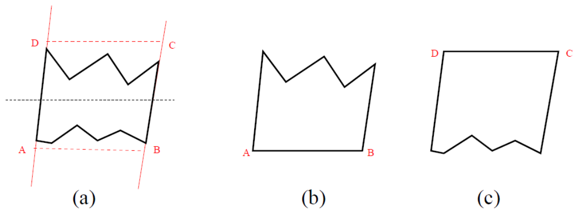
Removing divergencies in practice

- Using the U(1) theory. Remainder function

$$e^R = \frac{\langle W \rangle}{\left[\langle W \rangle_{U(1)}\right]^{\Gamma_{cusp}}}$$

Bern Dixon Smirnov

- Ratio of Wilson loops



$$e^r = \frac{\langle W \rangle \langle W_{square} \rangle}{\langle W_{top} \rangle \langle W_{bottom} \rangle}$$

Expansion for the remainder function

$$R \sim \int dp e^{ip\sigma} \left[C(p,\lambda) e^{-\tau \epsilon(p,\lambda)} - e^{-\tau} \Gamma_{cusp}(\lambda) C_1(p) \right]$$

Full theory

U(1) piece

If we computed r , using the second method, we would obtain just the first piece.

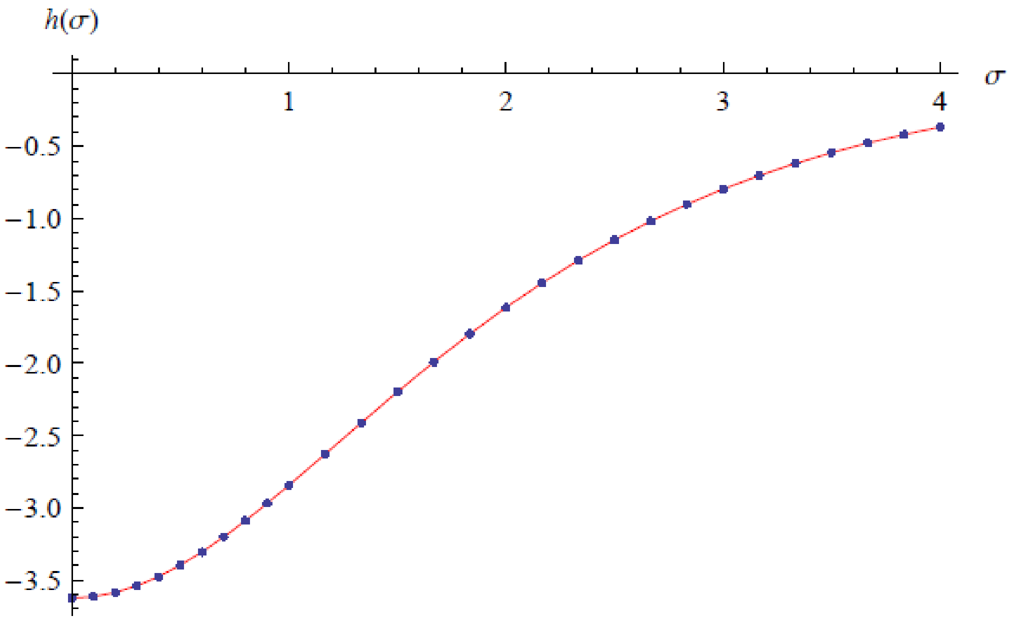
Checks

- Two loops

$$R \sim \int dp e^{ip\sigma} \left[C(p,\lambda) e^{-\tau \epsilon(p,\lambda)} - e^{-\tau} \Gamma_{cusp}(\lambda) C_1(p) \right]$$

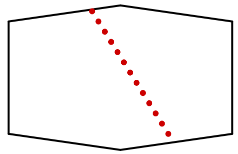
$$R \sim \lambda^2 e^{-\tau} \int dp e^{ip\sigma} \left[\tau \gamma(p) C_1(p) + C_2 - \Gamma_2 C_1(p) \right]$$

Term linear in τ is completely fixed.



Brandhuber, Heslop, Travaglini
Del Duca, Duhr, Smirnov
Zhang

Goncharov, Spradlin,
Vergu, Volovich



$$h(\sigma) = \int dp e^{ip\sigma} C_1(p)\gamma(p)$$

Belitsky, Gorsky,
Korchemsky

One loop anomalous dimension
One loop dispersion relation

$$h(\sigma) = \psi(\frac{3}{2} + i\frac{p}{2}) + \psi(\frac{3}{2} - i\frac{p}{2}) - 2\psi(1)$$

$$C_1(p) = \frac{1}{p^2+1}\frac{1}{\cosh\frac{p\pi}{2}}$$

Can be determined by a simple
computation in the U(1) theory.

Strong coupling results

Hexagon:

Only three cross ratios → all can be parameters of the expansion.

$$R = R_1 + R_{\sqrt{2}} + R_2 + \cdots$$

$$R_1 = -\cos\phi\, e^{-\tau}\left(\cosh\sigma \log[2\cosh\sigma] - \sigma\sinh\sigma\right)$$

$$R_{\sqrt{2}} = 4\cos\phi \int \frac{d\theta}{2\pi}\frac{1}{(\cosh 2\theta)^2} e^{-\tau\sqrt{2}\cosh\theta + i\sigma\sqrt{2}\sinh\theta}$$

Relativistic dispersion relation at strong coupling.

U(1) result. Is just

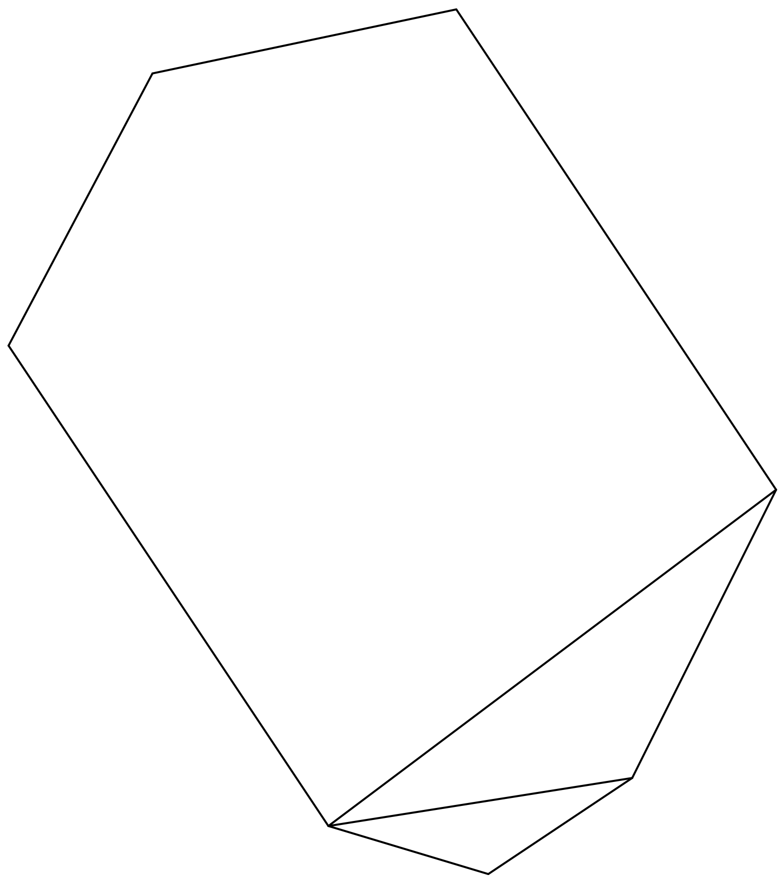
$$\int dp e^{ip\sigma} C(p)$$

Higher loop predictions

$$R \sim \cos\phi\, e^{-\tau} \tau^{L-1} \int dp\, e^{ip\sigma} C_1(p) \left[\gamma_1(p)\right]^{L-1}$$

More predictions are possible once we input the higher loop anomalous dimensions.

Building up a Polygon



3 parameters each time we add a line.

Wilson loops in general CFT's

- This OPE is valid for general conformal field theories in any dimension.
- In theories where the electric flux is conserved (For example, non-planar N=4 SYM and Wilson lines in the fundamental) the OPE will have similar properties.
- Similar one particle dispersion relation for twist one fields, but harder to compute.
- For ABJM one would probably have a similar behavior.

Conclusions

- The operator product expansion can be applied to Wilson loops.
- Divergences can be controlled and one has a manageable expansion.
- We have explicitly checked that the expansion works for 2 loops and also for strong coupling.
- We made predictions for larger values of the coupling.

Future

- Can this be used to determine the full correlator ? Bootstrap ?
- Need: better way to label the propagating states, to include multiparticle states, bound states, etc.
- Good news: we have the usual infinite set of charges !.

- by next meeting… ?